Design of Robust Fuzzy Controllers for Aerospace Applications

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Abstract

This paper details a Fuzzy - Feedback Linearisation controller applied to a non-linear missile. The design uses an evolutionary algorithm optimisation approach to a multiple model description of the airframe aerodynamics. A set of convex models is produced that map the vertex points in a high order parameter space (of the order of 16 variables). These are used to determine the membership function distribution within the outer loop control system by using a multi-objective evolutionary algorithm. This produces a design that meets objectives related to closed loop performance such as: rising time, steady state error and overshoot.

1. Introduction

The problem considered here is that of tracking a trajectory in the presence of noise and uncertainty. Many nonlinear analysis problems of engineering interest can be reduced to such a problem. Since the real system is not exactly the one used for the design, and since it is also subject to noise, the system will not follow the intended trajectory. Then the question of interest becomes: will the real trajectory, under the worst conditions possible, remain close enough to the nominal one. This could be defined as a robust trajectory tracking problem. Here, this kind of problem is addressed for a highly non-linear missile when the design of an autopilot is taken into account. Although such systems are well defined in terms of their dynamic behaviour, they have large uncertainty in their parameters and can cover large ranges of altitude and speed. By demanding small changes in system outputs, it is possible to exhibit the non-linear behaviour of the system which will then use a robust non-linear technique to achieve good performance.

The aim of this paper is to track the missile lateral acceleration demand in the presence of uncertainties introduced through the aerodynamic coefficients. The g demands are considered for both pitch and yaw planes, using the missile rudder and elevator as control surfaces hence yielding a system with 2 inputs and 2 controlled outputs.

It has been shown previously [1] that the desired tracking performance can be obtained by assuming an exact knowledge of aerodynamic coefficients and missile configuration parameters (i.e., reference area, Mach number, mass, moment of inertia) in the entire flight envelope. In practice however, this assumption is not valid and also, if there are either parameter variations or external disturbances, feedback-linearisation can no longer guarantee the desired performance(neither is robustness guaranteed).

Conversely fuzzy logic theory is useful when dealing with vague and imprecise information such as uncertain measurement values, parameter variations and noise [2]. This implies that a combination of an input/output linearisation technique (nonlinear control law) and a fuzzy logic trajectory controller have been suggested to be considered here. The design uses a GA optimisation approach to a multiple model description of the airframe aerodynamics. This is used to determine the membership function distribution within the outer loop control system by using a multi-objective GA that meets objectives related to closed loop performance such as: rising and settling time, steady state error, and overshoot.

2. HORTON Missile model

The missile model used in this study derives from a non-linear model produced by Horton of Matra-British Aerospace [3]. It describes a 5 DOF model in parametric format with severe cross-coupling and non-linear behaviour. This study will look at the reduced problem of a 4 DOF controller for the pitch and yaw planes without roll coupling. The angular and translational equations of motion of the missile airframe are given by:

$$\begin{aligned}
\dot{q} &= \frac{1}{2} I_{yz}^{-1} \rho V_o S d (\frac{1}{2} d C_{mq} q + C_{mw} w + V_o C_{m\eta} \eta) \\
\dot{w} &= \frac{1}{2m} \rho V_o S (C_{zw} w + V_o C_{z\eta} \eta) + U q \qquad (1) \\
\dot{r} &= \frac{1}{2} I_{yz}^{-1} \rho V_o S d (\frac{1}{2} d C_{nr} r + C_{nv} v + V_o C_{n\zeta} \zeta) \\
\dot{v} &= \frac{1}{2m} \rho V_o S (C_{yv} v + V_o C_{y\zeta} \zeta) - U r \qquad (2)
\end{aligned}$$

where the axes(x, y, z), rates(r, q) and velocities (v, w) are defined in (Figure 1).

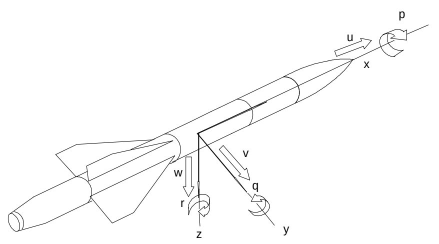

Figure 1. Airframe axes

Equations (1,2) describe the dynamics of the body rates and velocities under the influence of external forces (e.g. C_{zw}) and moments (e.g. C_{mq}), acting on the frame. These forces and moments are derived from wind tunnel measurements. The aerodynamic coefficients: C_{yv}, $C_{y\zeta}$, C_{nr}, C_{nv} and $C_{n\zeta}$ are presented by polynomials which are fitted to the set of curves taken from look-up tables for different flight conditions (roll angle 0° and 45°) shown in Table1 and Table2. The detailed description of the model can be found in [4].

The state-space form of the non-linear system of the home missile is written in a matrix form:

$$\begin{aligned}
\dot{x} &= f(x) + g(x)u \\
y &= h \\
&= \left[\begin{array}{c} h_1 \\ h_2 \end{array} \right] = \left[\begin{array}{c} x_1 \\ x_3 \end{array} \right] \qquad (3)
\end{aligned}$$

For the selected outputs (lateral velocities) an approximate input-output linearisation has been applied in our previous work [1]. A combination of neglecting sufficiently small terms during the differentiation process and proposing an output that is an approximation of the desired one has been used which has resulted in a linear equivalent system with no internal or zero dynamics.

The effect of neglecting small terms (the side-slip force acting on the control surfaces) in the g vector field is to eliminate a non-linear zero in the system within the model description, and which is not taken into account in the non-linear control design. It has been shown in [5] that provided the side-slip force is not too great this will not affect the performance of the control design in a significant manner.

The required static state feedback for decoupled closed loop input/output behaviour is given by [6] as:

$$u = E^{-1} \left\{ v - \left[\begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} \right] \right\} \qquad (4)$$

where E^{-1} is the decoupling matrix of the system and it is nonsingular.

After applying feedback linearisation technique the linearised closed loop system can be written as:

$$\ddot{y}_i = v_i \qquad (5)$$

where v is the new linearised system input.

It has been shown [1] that the desired tracking performance for lateral acceleration can be obtained by assuming an exact knowledge of aerodynamic coefficients and missile configuration parameters (i.e., reference area, Mach number, mass, moment of inertia). In practice however, this assumption is not valid and also, if there are parameter variations or external disturbances, feedback-linearisation can no longer guarantee the desired performance or neither is robustness guaranteed. For these reasons, a combination of an input/output linearisation technique (nonlinear control law) and a fuzzy logic controller(trajectory controller) have been chosen to be considered here.

3. Fuzzy trajectory controller

Figure 2 shows the non-linear controller structure. A fast linear actuator with natural frequency of 250 rad/sec has been included in the non-linear system. The fixed gains used in the design of the nominal model correspond to natural frequency $w_n = 50(rad/sec)$ and damping factor $\zeta = 0.7$ of the closed loop system. In this paper the gains are designed by using fuzzy set theory in order to deal with parametric uncertainties in the aerodynamic coefficients.

The trajectory controller has been designed based on fuzzy logic theory as a two input - one output system

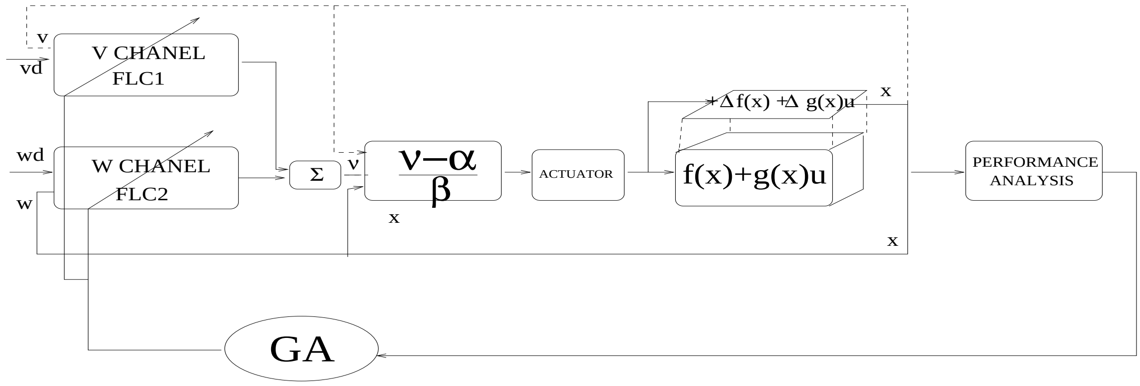

Figure 2. Trajectory control design

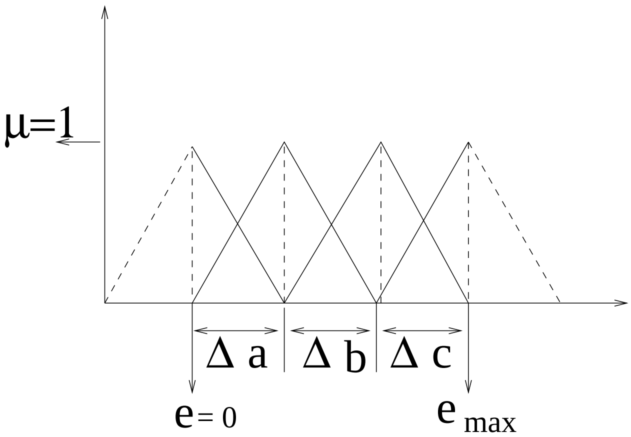

Figure 3. Membership functions set

with four membership functions for each variable (see figure 3). The membership functions position and the rules are generated using an evolutionary algorithm.

4. Multi-modelling: Sensitivity Analysis

Table 1 and 2 present the polynomials for the aerodynamic coefficients in supersonic range for different roll angles 0° and 45°. They are a set of curves in the plane of total incidence σ in $[rads]$ and Mach number M. In these tables the c_{yv} polynomials present the *normal force* curves, the x_{cp} present the *centre of pressure* curves, c_{yz} present the *rudder and elevator control forces* curves, and finally the c_{nr} present the *damping yawing and pitching moments* curves which are reasonably proportional to body rates.

The variations in aerodynamic coefficients have introduced parametric uncertainties into the non-linear system. A large excursion on perturbations within the whole range of aerodynamic roll angles 0° and 45° have been examined and perturbations on each of the aerodynamic coefficients $(c_{yz}, c_{yv}, x_{cp}, c_{nr})$ have been introduced into the system in a large variety of percentage deviation from nominal values.

Based up on simulations it has been found that some coefficients can be allowed larger percentage variation from the nominal case than others. Within the system we are able to tolerate $\pm 50\%$ uncertainty in c_{yz}, c_{yv}, c_{nr} before it goes unstable. Also it has been found that the centre of pressure x_{cp} and the control surfaces c_{yz} polynomials have most significant effect on the close loop performance (the system is very sensitive to small changes) while the damping moment contribution in c_{nr} is small and the system is almost insensitive so can be simplified to be independent aerodynamic roll angle.

The sign of x_{cp} can tell us whether the system is stable or not. When the σ term of x_{cp} is varied to around $+50\%$ change we get an unstable system.

Normal force	$c_{yv_0} = -25 + 1.0M - 60\sigma$
Control surfaces	$c_{y\zeta_0} = 10 - 1.6M + 2.0\sigma$
Centre of pressure	$x_{cp_0} = 1.3 + 0.1M + 0.2\sigma$
Damping moment	$c_{nr} = -500 - 30M + 200\sigma$

Table 1. Roll angle = $0°$

Normal force	$c_{yv_0} = -26 + 1.5M - 30\sigma$
Control surfaces	$c_{y\zeta_0} = 10 - 1.4M + 1.5\sigma$
Centre of pressure	$x_{cp_0} = 1.3 + 0.1M + 0.3\sigma$
Damping moment	$c_{nr} = -500 - 30M + 200\sigma$

Table 2. Roll angle = $45°$

5. GA operating on FLC membership functions

The proposed framework maintains a population of fuzzy rule sets with their membership functions and uses the evolutionary algorithm to automatically derive the resulting fuzzy knowledge base.

A hybrid real valued/binary chromosome has been used to define each individual fuzzy system. The real valued parameters are defined as being the $[\delta a\, \delta b\, \delta c]$ values shown in figure 3. The binary component encodes the set of rules used in the system. Each rule is either on or off (0/1) and corresponds to the form:

$$\text{if } A_i \text{ AND } B_j \text{ then } O_k \tag{6}$$

where A_i denotes membership function i of input A, B_j denotes membership function j of input b, and O_k denotes membership function k of the output O. This process allows a full set of rules to be developed for the fuzzy system, but maintains a fixed length chromosome. The four membership function structure leads to a chromosome with 9 real valued genes and 64 binary genes. The fuzzy system used product for the member function 'AND'. The 'OR' function was not required as the rules were all expressed as 'AND' terms. The implication method was to chose the minimum value and crop the output member functions. The aggregation method was to choose the maximum values of the set of member functions. A centroid approach was used to defuzzify the output.

The evolutionary algorithm[7] follows the usual format of ranking, selection, crossover, mutation and evaluation but with the real and binary parts of the chromosomes being processed separately. The same number of offspring are generated as parents and a total replacement policy is used. This helps slow convergence and helps to reduce the effects of the noisy objective functions. A multi-objective approach was used to identify good solutions. A method known as non-dominated ranking was used in the evolutionary algorithm to allow the multi-objective problem to be handled easily. A detailed description of the non-dominated ranking process may be found in [8].

Five objectives were used: rising time, steady state error, overshoot, settling time, and integral squared error. Two of the objectives, overshoot and rising time, have been treated as penalties in order to meet the specified requirements, i.e., if the parameters are within a required range, the penalty is zero; the penalty then increases when a threshold is exceeded.

To reduce the effects of the noise in the objective values, 5 trials of each chromosome were performed and the maximum values for each objective returned to the evolutionary algorithm. The evolutionary algorithm was run with a population size of 20 and for 300 generations.

6. Results

Figure 4 shows the fuzzy surface of the trajectory controller generated by the evolutionary algorithm. This has been developed with the model exercising the full range of aerodynamic coefficients defined by tables 1 and 2. The performance of the fuzzy controller was verified by 200 random trials and the results are summarised in figure 5, where the solid line shows the response for $0°$ roll angle, and the broken line is for $45°$.

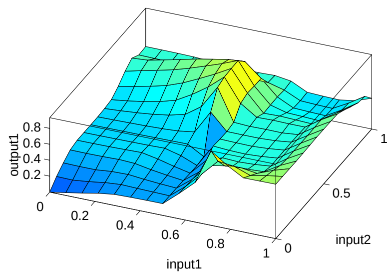

Figure 4. Surface of two input, one output fuzzy controller

The desired acceleration a_d is achieved by using the non-linear equation $a_d = f(v)$. Therefore the trajectory controller performs a desired acceleration as a function of the lateral velocity demand. The error dynamics are constructed using the a_d signal and the feedback of the actual states - velocity, rate, and acceleration. The results for lateral acceleration demand $10[m/sec^2]$ are shown in figure 5.

Figure 6 for fixed gains trajectory controller shows the influence on steady state error for both acceleration and velocity responces which is caused by the centre of pressure aerodynamic coefficient (x_{cp}) for the extreme case when roll angle is 45°. Since we are controlling indirectly lateral accelaration through velocity and the objectives on close loop performance are defined for the sideslip velocity, the steady state error on lateral acceleration for the extreme case roll angle 45° has not been corrected by the fuzzy trajectory controller.

However we have shown that the designed fuzzy controller can achieve very good desired tracking performance for sideslip velocity both cases roll angle 0° and 45° with no overshoot and almost no steady state error. The non-linear approach is also shown to be reasonably accurate, as the predicted and actual performances are very close.

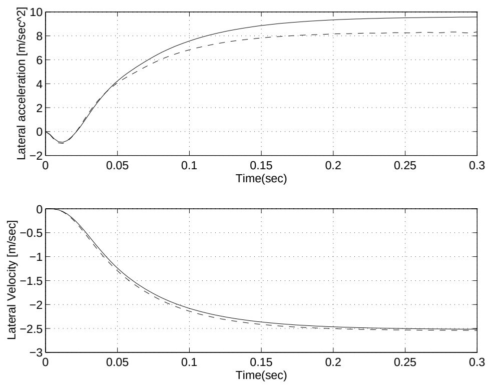

Figure 5. Acceleration and lateral velocity for $a_d = 10$, fuzzy trajectory controller

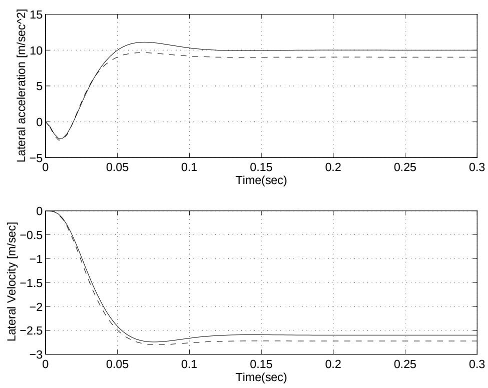

Figure 6. Acceleration and lateral velocity for $a_d = 10$, fixed gains

7. Conclusions

We have evaluated the robustness of feedback linearisation on the significant parametric uncertainty introduced into the system through the aerodynamic coefficients. We have proposed a fuzzy outer loop to improve the robustness. We have shown that the evolutionary algorithms can produce a good set of results that populate the Pareto solution set and we can also say for the multi-objective form provide a way of trading off one solution against the other. This particular solution can tolerate 50% variations before loosing good performance. This corresponds to the sensitivity analysis. These systems have very large range in dynamics and we have demonstrated that this technique can provide robust solutions.

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