EVOLUTIONARY GUIDANCE FOR MULTIPLE MISSILES

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Abstract: A new approach to guiding a swarm of missiles using an on-line evolutionary algorithm to optimise the flight paths is introduced. The missiles are flown via intermediate points which are adjusted dynamically by the evolutionary algorithm, then towards the point of impact with the target, thus altering the trajectory shape. Evolutionary algorithms are robust global optimisation techniques and are able to cope well with noise and uncertainty. This framework can be used to generate optimal flight profiles that satisfy multiple objectives and constraints. Example trajectories are presented for a multiple missile scenario using a highly non-linear model of a boost-glide missile.

Keywords: Data Fusion, Genetic Algorithms, Guidance Systems, Missiles, Multi-objective Optimisation

1. INTRODUCTION

High value and high threat targets are often defended well or difficult to intercept. The use of missiles with multi-spectral seeker systems can improve countermeasure rejection and also improve target parameter estimates, but at a price. Incorporating multiple seekers into one missile without compromising the seeker and airframe performance is a difficult and therefore expensive engineering problem. To improve the probability of intercepting the target, a salvo of missiles can be launched.

An alternative approach is to launch a salvo of lower cost missiles, each having only a single seeker. By adding communication capability to the missiles, sensor data shared within the salvo can be fused with data from sensors external to the salvo, allowing each missile to generate an estimate of the target's dynamics and position in space.

The missiles could be homogeneous where they all have the same seekers and performance characteristics and exploit spatial diversity to improve target estimates, or heterogeneous where the seekers and performance could all be different, giving both spatial and spectral diversity. The missiles could act as an intelligent salvo, sharing data to improve countermeasure rejection and to improve target parameter estimates, or better still could co-ordinate their flight paths in order to create a swarm.

In conventional guidance algorithms, data from the sensor is fed directly to the guidance algorithm, generating lateral acceleration demands which are fed to the autopilot, which moves the control surfaces accordingly. Thus missiles fired as a salvo will need to have different guidance algorithms if different flight profiles are required (Creaser and Stacey, 1999).

In evolutionary guidance, the missile is first flown via a sequence of one or more points in space, before flying towards a predicted impact with the target. The points in space are evolved to generate a flight profile that is an optimal solution to a set of objectives and constraints. With multiple missiles, the flight profiles can be evolved simultaneously, each flight profile being evolved while accounting for the intended flight paths of the other missiles. The flight profile modification process is illustrated in figure 1.

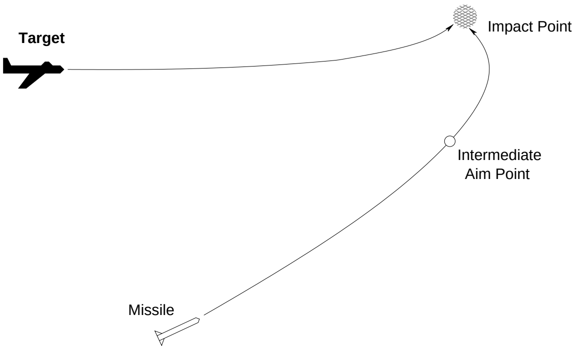

Fig. 1. Adaptation of flight path using intermediate points

Evolutionary guidance relies on the information from the seeker being fused with the estimate of the current missile position and orientation, and with any other sources of information on target and missile parameters, to give an estimate of the target position in space and its intentions. This data is then used to predict the target manoeuvre and therefore estimate a point of impact for the missile to fly towards.

With the evolutionary guidance approach, for most of the engagement there is no direct, deterministic path between the seeker and the autopilot (see figure 2). Thus the initial stages of the flight path can be independent of the target position and motion, allowing different trajectories to be generated easily. In this application, the scenario will be changing as the missile flies, therefore changing the objective functions. With a highly non-linear missile, there are likely to be many sub-optimal flight paths. Evolutionary algorithms are global optimisation techniques and are robust to noise in the objective functions and have been shown to be well suited to the single missile path planning problem (Quing *et al.*, 1997).

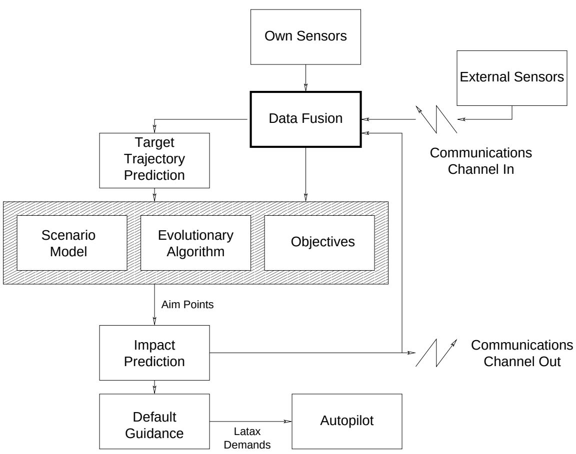

Fig. 2. Block diagram of Evolutionary Guidance process

The following sections first describe typical guidance heuristics, then the evolutionary algorithm. The missile model used in the examples is detailed and example results presented. Finally the paper concludes and indicates areas of future research.

2. GUIDANCE HEURISTICS

Proportional Navigation (PN) (Zarchan, 1997) has been used for many years and is well proven as a guidance algorithm. The main essence of the technique is to form a collision triangle, based on target position and velocity, and use it to estimate an impact point where the missile could first intercept the target. The missile then flies towards the impact point, rather than at the target and uses seeker angle rate to derive the lateral acceleration required to correct the position of the estimated impact point.

The impact point and required flight direction are implicit within the PN formulation and are not calculated explicitly. The process of estimating an impact point can be generalised to any predicted target manoeuvre, where the minimum time trajectory for the missile to fly is a straight line to the impact point. Proportional navigation has been enhanced to achieve this, such as Augmented Proportional Navigation (APN) (Zarchan, 1997) where target lateral acceleration is taken into account.

For a generalised minimum time guidance heuristic, once the impact point is estimated, the lateral acceleration required to steer the missile towards the impact must be calculated. This is achieved by calculating the angle between the current flight direction of the missile and the direction towards the estimated impact point. The missile must turn through this angle in the shortest time possible, given the current maximum lateral acceleration of the missile. Thus the angular rate required may be established and along with the relationship $a = v\omega_{max}$, where v is the forward velocity and ω_{max} is the maximum angular rate to be applied for the shortest time, the lateral acceleration and the duration of the acceleration event may be generated.

A sub-optimal approach where a lower acceleration is applied for a correspondingly longer time may also be used. This is closer to the operation of traditional PN based techniques. This sub-optimal technique is useful in missiles with high levels of sensor noise and lift-drag coupling, where many large course corrections can have an adverse effect on missile velocity.

Figure 3 shows the engagement geometry for a constant velocity target. The missile has a current velocity $\vec{V}_m\prime$ and must pull lateral acceleration in order to obtain a velocity $\vec{V}_m$. The missile must turn through angle θ, and forms the basis of the guidance heuristic described above. The projected point of impact, P_I, is calculated using (1), where t is the predicted impact time, τ is an arbitrary time during the engagement and V_m is the known scalar speed of the missile.

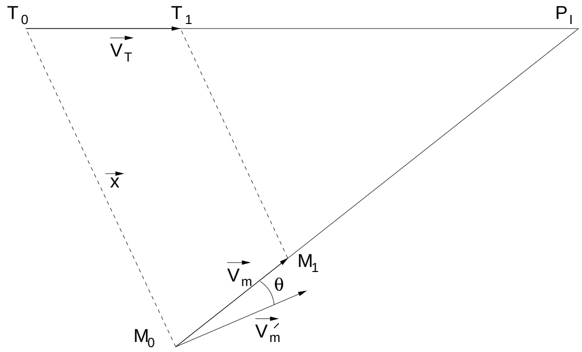

Fig. 3. Engagement geometry

$$\vec{x} = (T_0 - M_0)$$
$$M_\tau = T_0 + \tau \vec{V}_t - \frac{\vec{x}(t-\tau)}{t}$$
at $\tau = 1,$
$$V_m = |M_1 - M_0|$$
$$M_1 = T_0 + \vec{V}_t - \frac{\vec{x}(t-1)}{t}$$
$$\therefore M_1 = \vec{V}_t + M_0 + \frac{\vec{x}}{t}$$
$$\therefore V_m = |\vec{V}_t + \frac{\vec{x}}{t}|$$
$$\therefore \quad 0 = \frac{\vec{x}\cdot\vec{x}}{t^2} + 2\vec{V}_t \cdot \frac{\vec{x}}{t} + (\vec{V}_t \cdot \vec{V}_t - V_m^2)$$
$$\therefore \quad \frac{1}{t} = \frac{-\vec{V}_t \cdot \vec{x} \pm \sqrt{(\vec{V}_t \cdot \vec{x})^2 - (\vec{V}_t \cdot \vec{V}_t - V_m^2)\vec{x}\cdot\vec{x}}}{\vec{x}\cdot\vec{x}} \tag{1}$$

As we always want the shortest impact time, $1/t$ must be as large as possible, therefore (1) may be modified and the impact time may be calculated from (2).

$$q = (\vec{V}_t \cdot \vec{x})^2 - (\vec{V}_t \cdot \vec{V}_t - V_m^2)\vec{x}\cdot\vec{x}$$
$$t = \frac{\vec{x}\cdot\vec{x}}{\sqrt{q} - \vec{V}_t \cdot \vec{x}} \tag{2}$$

If q or t are negative, the missile is flying too slow and will never catch the target.

3. EVOLUTIONARY ALGORITHM

Evolutionary Algorithms are designed to mimic the natural selection process through evolution and survival of the fittest (Michalewicz, 1996; Goldberg, 1989). A *population* of M independent individuals is maintained by the algorithm, each individual representing a potential solution to the problem. Each individual has one *chromosome*. This is the genetic description of the solution and may be broken into n sections called *genes*. Each gene representing a single parameter in the problem, therefore a problem that has five unknowns for example, would require a chromosome with five genes to describe it.

The three simple operations found in nature, natural selection, mating and mutation are used to generate new chromosomes and therefore new potential solutions. In this paper, new chromosomes were generated by applying Gaussian noise, with a standard deviation that evolved along with each gene, to each gene in each chromosome. Each chromosome is evaluated at every generation using an *objective function* that is able to distinguish good solutions from bad ones and to score their performance. With each new generation, some of the old individuals die to make room for the new, improved offspring. Despite being very simple to code, requiring no directional or derivative information from the objective function and being capable of handling large numbers of parameters simultaneously, evolutionary algorithms can achieve excellent results.

In a command-guided swarm, a potential solution would be a matrix containing the intermediate aim point vectors for each of the missiles. The initial population of solutions is usually generated entirely at random, within some bounds to ensure that most (but not necessarily all) of the solutions are feasible to fly. If heuristics exist to find good sets of aim points, these could be used to help generate the initial population.

All the sets of solutions are evaluated by simulating the missiles and the target and establishing the overall performance of the trial solution. A set of objectives and constraints are used to guide the optimisation process; for example, to minimise the longest flight time, minimise the difference between the longest and shortest flight (simultaneous time-on-target), maximise the smallest difference between the impact angles etc. If a chosen intermediate aim point causes a missile to miss the target in the simulation, the guidance for the particular missile can default to fly straight towards the closest impact point.

These results will form a *Pareto Optimal Set* (Goldberg, 1989, Pages 197–201) where no single solution is better than any of the others when all objectives are taken into account. For example, trajectories with long flight times are undesirable but can achieve a very wide spread of impact angles, while short flight time trajectories will have a small range of impact angles but will have higher impact velocities.

Equation 3 was used to combine the multiple requirements into a single objective for the algorithm to process. For each trial solution, i, the final objective value to be minimised, $\mathcal{O}(i)$, is selected as the worst weighted performance objective, $f_n(i)$. This has the effect of trying to balance each of the weighted functions, with the weights corresponding to relative importance between the different performance measures $f_n(i)$.

$$\mathcal{O}(i) = \max(w_1(i)f_1(i), w_2(i)f_2(i), \ldots) \tag{3}$$

For the results shown in section 5, an evolutionary strategy with a base population of $M = 50$ trial solutions was used. At each generation, $N = 150$ new trial solutions were created, and all 200 evaluated. The best 50 were then chosen for the next generation. Real valued intermediate crossover was used to recombine the genes with a crossover rate of 10%. Gaussian mutation was applied to each gene at a rate that evolved along with each gene.

The best performing solution is selected and used to supply the aim points for the missiles. This approach is a little crude, as it can be seen in the results that the population forms into clusters around points in space that present good solutions. By always taking the best solution, the aim point can wander within the cluster, causing latax demands to be generated. A clustering algorithm could be used to identify the centre of the best performing cluster of individuals, therefore making the aim point more stable.

If a missile passes an aim point then it switches out the evolutionary algorithm and heads for the point of impact with the target. In the simulation, when one missile comes within one kilometre of the target, all the missiles change if necessary to aim at the target and the guidance heuristic, along with any fused target position information, is used to guide the missile.

4. MISSILE MODEL

The model is based loosely on a ship-launched boost-glide missile. For simplicity and to increase processing speed, simple single step integration with a coarse 0.1 second interval was used for the main missile models, and a 0.3 second interval for the flight path projection simulations. This approach meant the integration was too coarse to allow the missile control system to be modelled, so the performance characteristics were generated by modelling the limits imposed by the body aerodynamics. The missile is restricted to a two dimensional engagement in the vertical plane and is subject to the initial boost force, changing mass, gravity, forward drag, lift-drag coupling, changes of air density and speed of sound with altitude.

Equation 4 details the calculation for the lateral acceleration demand of the missile. The equation calculates the acceleration needed to steer to aim towards the impact point, and also calculates the acceleration needed to correct for the effects of gravity. The angle to steer, θ, is as defined in figure 3; δt is the time step of the model; $|\vec{V_m}|$ is the missile speed; $\hat{\vec{V_n}}$ is the unit vector in the direction that the lateral acceleration must be applied; and k is a constant that acts to damp the response of the missile. In the trials shown in this paper, a value of $k = 5$ was used to help prevent many rapid course corrections causing excessive drag on the missile.

$$l_d = \frac{|\vec{V_m}|\theta}{k\delta t} + \hat{\vec{V_n}} \cdot [0,\ 9.8] \qquad (4)$$

The lateral acceleration demand, l_d, may not be achievable though and so must be limited by calculating the maximum possible demand for the given conditions as shown in (5).

$$l'_d = \begin{cases} \min(|l_d|, l_{max}) & l_d \geq 0, \\ -\min(|l_d|, l_{max}) & l_d < 0, \end{cases} \qquad (5)$$

Figure 4 shows the non-linear relationship used to determine the maximum latax demand, l_{max}, for a given speed. If the required latax exceeds the maximum value, the demand is cropped. The graph shows the performance for the missile in the glide phase where the mass is at the minimum value and the missile at sea level.

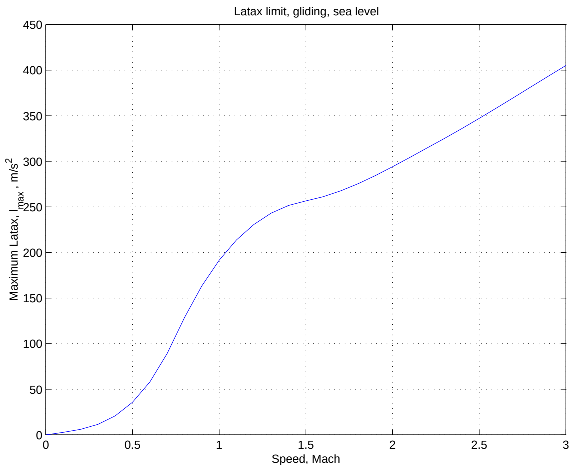

Fig. 4. Curve of maximum latax with respect to forward speed

Equations 6, 7 &8 give the approximations for the speed-of-sound, V_s, air density, ρ, and mass m. Speed of sound and air density vary with respect to altitude, h.

$$V_s = 340.3 - 0.0041\,h \qquad (6)$$

$$\rho = 1.375\ \exp\left(\frac{-h}{11000}\right) - 0.150 \qquad (7)$$

$$m = 34(\tanh(1.5 - t) + 1) + 75 \qquad (8)$$

The forward acceleration, a, is calculated using (9), boost force using (10), change in forward velocity using (11), and change in position with (12).

$$a = \frac{f - \frac{1}{2}\rho|\vec{V_m}|^2 A C_d}{m} - \left|\frac{l'_d}{C_{ld}}\right| \qquad (9)$$

$$f = 17000(\tanh(5 - 2t) + 1) \qquad (10)$$

$$\delta\vec{V_m} = \delta t\left(a\hat{\vec{V_m}} + [0, -0.98] + \hat{\vec{V_n}} l'_d\right) \qquad (11)$$

$$\delta P = \delta t \vec{V_m} \qquad (12)$$

A missile cross-sectional area of $A = 0.0254$, mean drag coefficient of $C_d = 0.45$ and mean lift-drag ratio of $C_{ld} = 3.5$ were used in the simulations. The actual values for C_d and C_{ld} were different for each missile by up to $\pm 10\%$.

5. RESULTS

A sample swarm with four missiles, launched from coordinate $[0, 0]$, was simulated engaging a target flying with constant velocity at Mach 1. The missile sensor system was simulated by corrupting the exact target position and velocity with noise, causing the estimated impact point to wander. An evolutionary algorithm with a working population of 50 trial aim-points was used, with one generation being simulated as 0.1 seconds.

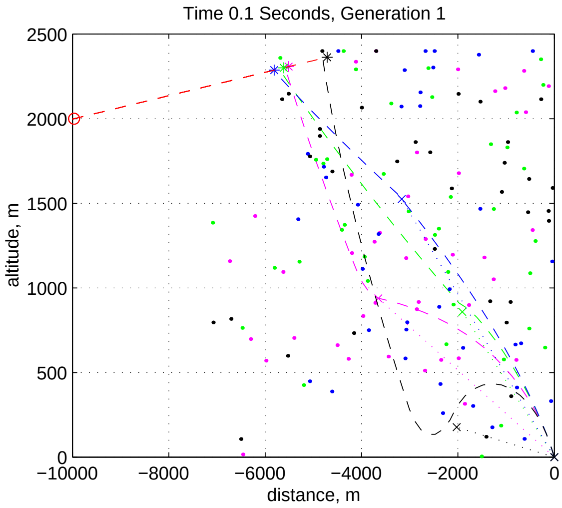

Fig. 5. Initial predicted trajectories showing location of fixed points in evolutionary algorithm population at generation 1

Three objectives were optimised in the simulation; maximise lowest latax limit of the four missiles at impact, minimise longest flight time, and maximise the smallest difference between impact angles. The three different objectives were combined using (13) to give a single value to minimise.

$$\mathcal{O} = \max\left(\frac{1}{2}\left(l_{\underset{\max}{PN}} - \min_{n=1\ldots4}\left(\underset{\max}{l_n}\right)\right), \; 5\left(t_{PN} - \min_{n=1\ldots4}(t_n)\right), \; 4\delta\alpha \right) \quad (13)$$

In (13), $l_{\underset{\max}{PN}}$ is the maximum latax at impact for a PN based missile, t_{PN} is the flight time for a PN missile, t_n is the flight time of missile n, $\underset{\max}{l_n}$ is the maximum latax at impact of missile n, and $\delta\alpha$ is the smallest difference between impact angles with the target.

Figure 5 shows the initial population at instant of launch. The missile velocity is still zero at this instant, so a number of generations can be executed before a significant speed has been attained. The missile positions are marked with crosses, the proposed fixed points are dots, the target position is a circle, the predicted trajectories are dashed lines, and the predicted impact points are stars.

Figure 6 shows the state at 2.5 seconds (25 generations). Here the missiles are approaching maximum velocity, and aim-points are forming tight clusters. At 6 seconds (figure 7) the clusters are tight and the trajectories are forming well.

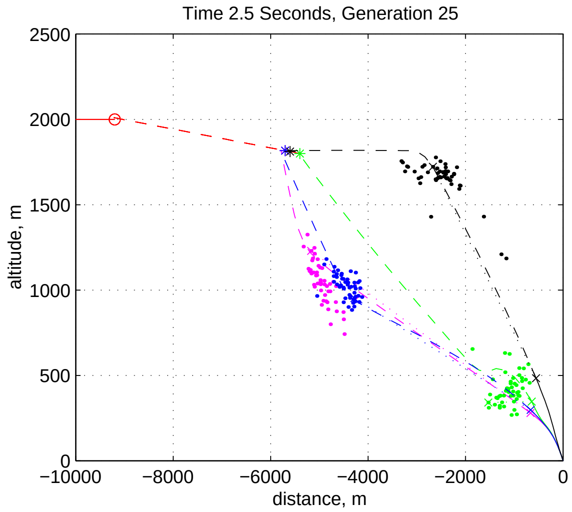

Fig. 6. Partial and predicted trajectories showing location of fixed points in evolutionary algorithm population at generation 25

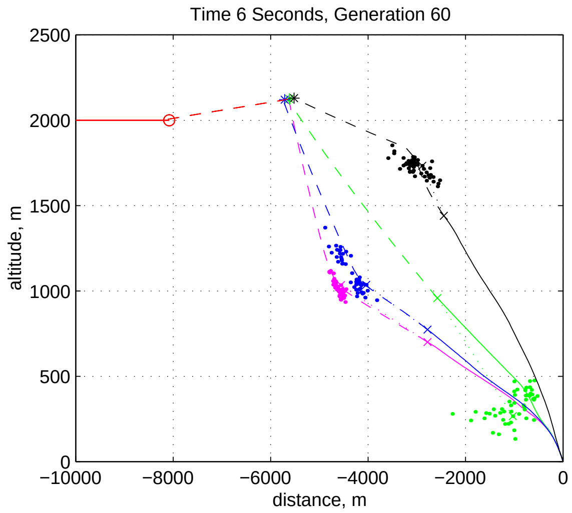

Fig. 7. Partial and predicted trajectories showing location of fixed points in evolutionary algorithm population at generation 60

The first impact occurs at 13.2 seconds, and is shown in figure 8. It can be seen that one missile took a nearly direct route to the target, while the other missiles took a longer course in order to alter their impact angle with the target. The smallest difference between the impact angles was 13.6°.

Figure 9 shows the velocity profiles of the four missiles. It is clear that the different manoeuvres contributed to the drag, slowing the missiles down. The interception occurred near the limit of the missiles' ranges, with the missiles travelling at about Mach 1 at impact despite reaching a peak of Mach 2. The missile that appeared to take the shortest path did not reach

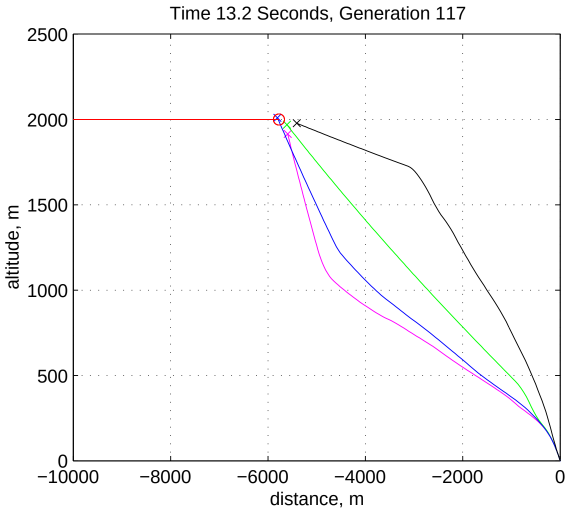

Fig. 8. Final trajectories, first missile impact at 13.2 seconds

the target first. This is because the missile manoeuvred heavily in the early stages of the flight and its velocity was reduced.

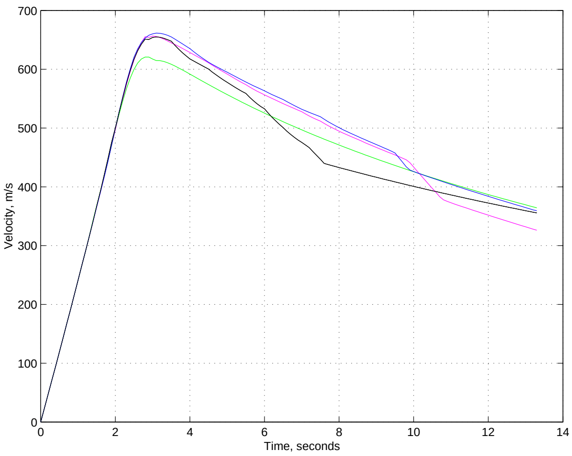

Fig. 9. Velocity profiles of the four missiles

6. CONCLUSIONS

The evolutionary guidance approach provides a comprehensive framework allowing multiple missiles to coordinate attacks on single or multiple targets. The framework also allows for data from multiple sensors to be fused easily, as the guidance requires estimates of missile and target positions etc. in absolute coordinates to be used. The results were generated using a highly non-linear missile, combined with noisy measurements and uncertain system models. This shows clearly that the method is tolerant of complexity and many sources of error.

The technique does however require significant processing resources. On todays fast machines, evolutionary guidance as demonstrated will be just realisable in real time, albeit with relatively small population sizes. The larger the population size used, and the more accurate the missile simulations, the better the guidance will perform.

Future work will include missiles with bang-bang control to reduce missile cost, applying clustering algorithms to the aim-points to give more consistent steering demands, and implementing target motion predictors other than constant velocity flight.

7. ACKNOWLEDGEMENTS

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