Arbitrary Pulsed Radar Waveform

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***Abstract*—This paper describes the design of a very short pulsed radar waveform in which all the pulsed parameters may be designed so as to support pulse Doppler operation yielding an ambiguity free measurement of target range and velocity whilst maintaining good target visibility at all ranges/velocities in the presence of clutter. We imagine the coherent processing interval to be comprised of a sequence of pulses whose inter-pulse timings and pulse widths can be chosen independently for optimal performance. The waveforms derived here demonstrate an interesting alternative to conventional low, medium or high PRF processing.**

I. Introduction

In the field of pulsed radar waveforms, the pulse repetition frequency (PRF) falls into three regimes [1]:

Low PRF. These are sufficiently low so as to avoid range ambiguities, however, velocity data is likely to be highly ambiguous. Low PRF affords very little ability to reject clutter, particularly from fast moving platforms since main beam clutter (MBC) and its ambiguous repetitions in the Doppler domain overwhelm target returns at all Doppler frequencies. Consequently, low PRF waveforms are not viable for airborne pulse Doppler applications but they are ideally suited to ranging applications when little or no clutter is present such as look-up scenarios or at ranges which are less than the altitude of the radar platform.

High PRF. A high PRF waveform is one which is sufficiently high so as to avoid Doppler, and hence velocity, ambiguities, however, range data is likely to be highly ambiguous. This waveform affords good rejection of MBC but suffers from side lobe clutter (SLC) which becomes spread between $\pm V_P$, where V_P is the ground speed of the platform. Consequently, high PRF is ideally suited to velocity measurement applications. Since also a large number of pulse returns may be received in each coherent processing interval (CPI), large processing gains are possible leading to good long range detection performance. Fast moving targets may be detected outside the bandwidth of SLC, however, high PRF struggles with slow moving targets due to the SLC and its multiple ambiguous repetitions across the receiving period.

Medium PRF. Medium PRF is ambiguous in both range and Doppler (velocity). It is seen as a compromise between high and low PRF since it avoids the severe problems of MBC experienced by low PRF and the SLC related problems of high PRF. Medium PRF subdivides the beam dwell time into several CPIs (typically 8); each transmitting a different PRF, in order to decode the true range and velocity of targets and to ensure all round target visibility in the face of clutter. In order to accommodate multiple CPIs, each CPI duration must be limited which, in turn, leads to more modest processing gains and detection ranges.

The current study seeks to design a waveform which achieves the best of all three PRF regimes described above, but using a much shorter CPI. This has been pursued along two avenues of enquiry.

1. Through the design of pulse timings based on standard sequences such as Fibonacci series, doubling sequences and Costas codes.
2. Through the use of an evolutionary algorithm to evolve the PRIs and pulse widths.

This paper is arranged as follows. Section II gives the theory describing the eclipsing of pulse sequences and of relevant aspects of the radar processing. The derivation of pulse sequences based on standard coding strategies is described in section III and in section IV the evolutionary approach to pulse sequences optimisation is described. The results are presented and discussed in section V and, finally, section VI draws some conclusions.

II. Theory

The design aim is to derive waveforms which yield an ambiguity diagram as close to the ideal as possible, whilst trading ambiguities against eclipsing losses. The quality of all the waveforms designed by either method is compared. A general pulse sequence of N pulses is illustrated in Figure 1, in which the pulse widths are $\tau_1, \tau_2, \ldots \tau_N$, the PRIs are $T_1, T_2, \ldots T_N$ and the times of the rising edges of the pulses are $t_1, t_2, \ldots t_N$. The first pulse is always taken to start at zero time ($t_1 = 0$).

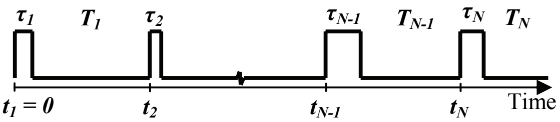

Figure 1 Timings of general pulse sequence

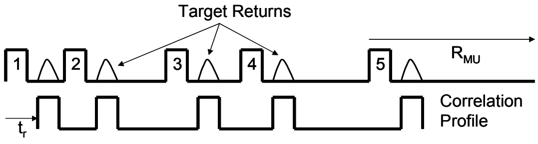

Figure 2: Range bins chosen for correlation at time offset t_r and corresponding target returns for an example 5-pulse waveform.

The individual PRIs may be chosen to be ambiguous in range or velocity, however to guarantee that the furthest target of interest has been illuminated by all of the pulses, the last PRI of the pulse sequence determines the maximum unambiguous range, i.e.:

$$R_{mu} = \frac{cT_N}{2} \tag{1}$$

Although the individual pulse timings could be adjusted independently, for this study we have considered a case where the individual pulses are modulated and then compressed on reception. Once pulse compression has been applied, a Discrete Fourier Transform (DFT) is then applied to extract the range and velocity of any targets present in the scene.

A. *DFT Processing*

Target extraction is performed by correlating the transmitted pulse train with the received echo data at each possible range offset of interest as shown in Figure 2. If each pulse has a different length and modulation scheme (e.g. some pulses may use a linear frequency sweep for compression, others may be phase coded) then the correlation process may be applied in its full form and both pulse compression and ambiguity resolution may be achieved in a single processing step. If the pulses are all the same length and have the same modulation, conventional pulse compression may be applied first, reducing the processing required for the pulse-train correlation step as only the range-bins where pulses are expected need to be included in the correlation process.

Once the initial pulse train correlation has been performed, the pulse train is Doppler shifted and the correlation process repeated, now extracting targets moving at the intended Doppler velocity. Thus an entire range/Doppler map may be formed and the targets subsequently detected. Alternatively, the range cells may be offset by each of the PRIs and 'stacked' and Finite Impulse Response (FIR) filtering applied as used in classic Moving Target Indication (MTI) systems [5].

The DFT process of pulse-compressed returns may be described as in (2)

$$r_{R,v} = \left| \sum_{m=1}^{N} E(t_m + \frac{2R}{c}) \exp\left(j2\pi f_0 (1 + \frac{2v}{c}) \right) \right|, \tag{2}$$

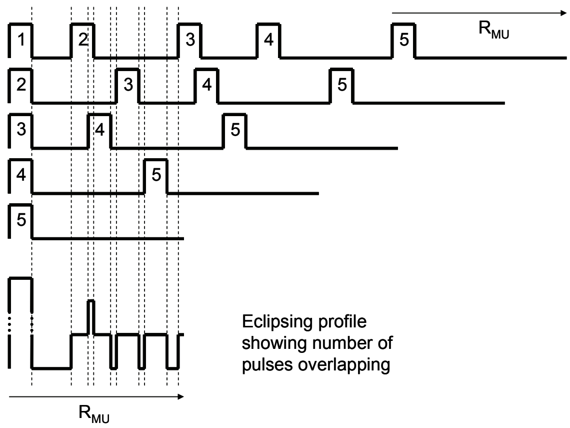

Figure 3: Generation of Eclipsing profile for example 5-pulse waveform.

where $r_{R,v}$ is the magnitude of the complex integrated output corresponding to R at velocity v, $E(t)$ is the complex echo voltage at time t after pulse compression, t_m is the transmission time of pulse m, c is the speed of propagation and f_0 is the transmission frequency. Note that (2) copes with unequal time sampling given by the values of t_m used by the code (as opposed to standard DFT techniques which assume a fixed PRI).

B. *Eclipsing*

The number of pulses eclipsed at any range R may be calculated as in (3), where n_R is the number of pulses eclipsed at range R, t_m and t_k are pulse transmission start times of the N pulses, τ_k is the length of pulse k and c is the speed of propagation. The summations are a summation of Boolean values, given the logical outcome of the inner expression. The summation process is shown graphically in Figure 3.

$$n_R = \sum_{k=1}^{N-1} \sum_{m=k}^{N} \left((t_m - t_k) \le \frac{2R}{c} \le (t_m - t_k + \tau_k) \right) \tag{3}$$

In order to minimise eclipsing, the timing of any of the received pulses must not coincide with the transmission of pulses in the sequence. Thus:

$$t_r(k) = t(k) + t_d \ , \tag{4}$$

in which $t_r(k)$ is the reception time of the k^{th} pulse and t_d is the range delay. The timing of the k^{th} transmitted pulse is given by the sum of all previous PRIs:

$$t(k) = \sum_{m=1}^{k-1} T_m \ , \tag{5}$$

giving

$$t_r(k) = \sum_{m=1}^{k-1} T_m + t_d \ . \tag{6}$$

An unavoidable condition which definitely results in one of the received pulses coinciding with one of the transmitted pulses is when $t_d = t(k)$, for any $k = 1, 2, \ldots N$. The objective is to deduce pulse timings for which no other received pulses coincide with transmitted pulses for any other values of t_d (or at least as few coincidences as possible are incurred).

Coincidences *will* occur when $t_d = t(k)$ but are not confined to just these cases. In other words, one wishes to ensure:

$$t_r\left(k^{/}\right) \neq t(k), \tag{7}$$

where $k^{/} = 2, 3, \ldots 8$ and $k = 1, 2, \ldots .8$

Note, this only guarantees the absence of large eclipsing losses for the case of $t_d = t(k)$, no such guarantee be made for other values of t_d.

The condition in (7) is met when the time of one transmitted pulse plus the time of another transmitted pulse does not equal the time of yet another transmitted pulse, i.e.:

$$t_r\left(k^{/}\right) = t\left(k^{/}\right) + t\left(k^{//}\right) \neq t(k) \tag{8}$$

where $k^{//} = 2, 3, \ldots .8$ and $k^{/}$ is not necessarily equal to $k^{//}$. The inequality of (8) can be assured if:

$$t(k) > t(k-1) + t(k-2) \tag{9}$$

From (5), this requires:

$$\sum_{m=1}^{k-1} T_m > \sum_{m=1}^{k-2} T_m + \sum_{m=1}^{k-3} T_m \tag{10}$$

The conditions defined by (8) to (10) must also include a margin of inequality to allow for the transmitted pulse width. The constraint given by (9) may be met by a sequence of pulse timings based on a Fibonacci series. Furthermore, the constraint described by (8) may also be met for other timing sequences such as successive doubling and Costas codes.

A similar analysis can be offered for the minimisation of velocity blindness by inverting timings into the (Doppler) frequency space. Now, the margin of inequality must allow for the bandwidth of the MBC rejection filter.

III. Pulse Timings Based on Standard Sequences

A. Design Goal

Initially, the design of pulse timings has been investigated for the minimisation of eclipsing losses. Pulse sequences have been designed by selecting pulse repetition intervals (PRI = 1/PRF) according to various coding strategies. The eclipsing losses of any pulse sequence can be quantified in terms of the worst case eclipsing loss (L_E) or its worst case range visibility ($P_R = 1 - L_E$) and the total range blindness (which is the aggregated eclipsing loss for all ranges) and is equal to the total duty ratio (i.e. the ratio of the sum of all pulse widths to the sum of all the PRIs).

B. Fibonacci Series

A nine pulse sequence has been derived ($N = 9$) for a common pulse width of 7μs ($\tau_1 = \tau_2 = \ldots \tau_N = 7\mu s$) and with pulse timings based on a Fibonacci series. Strictly, a Fibonacci series is defined as:

$$F_K = F_{K-1} + F_{K-2}, \tag{9}$$

in which the first two terms in the series are set to unity, i.e.:

$$F_1 = F_2 = 1.$$

In this work, a slight adaptation is made in which the pulse timings are derived from:

$$t_M = t_{M-1} + t_{M-2} + \tau . \tag{10}$$

The addition of τ onto the sequence minimises eclipsing by staggering each PRI by an additional pulse width. The choice of the first two PRIs is somewhat arbitrary and the last PRI is determined by the required unambiguous range (100km is used). It was further assumed that a maximum duty ratio of 10% would be a practical limit and so the lowest PRI (always T_1) would be set to the limit of 70μs. T_2 was somewhat arbitrarily set to:

$$T_2 = 2T_1 + 3\tau.$$

The full sequence is:

$T_{1,\ldots.9}$ = 70, 161, 77, 238, 315, 553, 868, 1421, 667 μs

C. Successive Doubling Series

Previously, the condition given by (8) could be satisfied by (9), which gave rise to pulse timings based on a Fibonacci series. However, this assumes that $k^{/} = k-1$ and $k^{//} = k-2$ and is therefore only valid when $k^{/} \neq k^{//}$. In some cases $k^{/} = k^{//}$. To cater for this eventuality requires that:

$$t(k) > t(k-1) + t(k-1)$$

i.e.:
$$t(k) > 2.t(k-1) \tag{11}$$

Timings derived on the basis of (11) express a successive doubling sequence in which the choice of the first PRI (but not the second one) is arbitrary and the margin of inequality must once more allow for the transmitted pulse width.

A nine-pulse sequence has been derived ($N = 9$) for a common pulse width of 7μs ($\tau_1 = \tau_2 = \ldots \tau_N = 7\mu s$) and with pulse timings based on:

$$t(k) = 2.t(k-1) + 15\mu s, \tag{12}$$

in which the 15μs accommodates the transmitted pulse width of 7μs plus an additional 8μs range blindness margin. The initial choice of the first PRI is, $T_1 = 100$ μs. The full sequence is given by (again assuming R_{mu}=100km):

$T_{1,\ldots.9}$ = 100, 115, 230, 460, 920, 1840, 1680, 7360, 667 μs

D. Costas Coded Series

The unique properties of Costas codes make them popular in secure frequency hopping sequences. Costas codes also boast low time side lobes and ambiguities. In the context of frequency coding, a Costas code is defined as [2]:

$$f_{k+i} - f_k \neq f_{j+i} - f_j, \tag{13}$$

in which k, j and i are all positive integers. This defines frequencies such that the difference between a pair in the sequence separated by any number, i, is not equal to the difference between any other pair separated by the same number, i, in the sequence. Pulse timings derived on the basis of (13) would appear to meet the eclipsing conditions of (8).

The following PRIs of an eleven pulse sequence (first 10 PRIs are based on Costas code) adhere to a Costas code: 140, 280, 560, 350, 700, 630, 490, 210, 420, 70, 667 μs. The transmitted pulse width was fixed at 7μs.

IV. Pulse Timings Identified Using Evolutionary Algorithms

A. Previous Medium PRF Solutions

Previous work [1] has sought to optimise the values of PRFs in a 3 of 8 medium PRF (MPRF) schedule for minimal range/velocity blindness in the presence of clutter. A near - optimal solution was found to be PRI = 63.11, 69.97, 77.07, 81.31, 90.06, 99.90, 109.75 and 119.00 μs for a fixed transmitted pulse width of 7μs. Although for MPRF use, each PRI would be used with a burst of pulses, the timings themselves have been tested for use with a 9-pulse system, where a last PRI of 667 μs has been added to ensure R_{mu}=100km.

B. Evolutionary Optimisation of Waveform

As previous work [1] had indicated that Evolutionary approaches are capable of identifying good solutions to the highly combinatorial problems of PRI optimisation, an evolutionary approach was developed for the optimisation of the arbitrary waveforms.

Evolutionary Algorithms (EAs) are optimisers which use an approach inspired by the natural phenomenon of biological evolution [3] and directly exploit the Darwinian concept of `survival of the fittest' where the best specimens of a species live long and produce many offspring while the weaker members of the population die young and have few or no offspring. Successive generations tend to become dominated by the best features from previous generations.

In the context of mathematical optimisation, each member of a population is a potential solution to the optimisation problem, which in this paper is a receiver search strategy. The `fitness' (quality) of each population member is measured by an `objective function', which in this paper is to minimise the velocity ambiguity of the waveform whilst also limiting the effects of range eclipsing and also minimising the total dwell time required from the waveform.

EA optimisation is an iterative process with each iteration representing one `generation'. Each generation should comprise members that are generally fitter than their predecessors as a result of selective breeding and replacement. As the population becomes fitter as a whole, the individual members will begin to appear very similar to one another until the population eventually converges to an optimal solution.

We found that the best results were obtained using an `Evolutionary Programme' (EP) [3]. In the terminology of the EP, each population member is called a `chromosome' which comprises a number of individual `genes'. In this paper, a chromosome represents the PRI sequence of a transmitted waveform and each gene represents a PRI in μs. The optimisation process performed by the EP is described by [4]

1. Generate an initial population of random chromosomes,
2. evaluate the fitness of each member of the population,
3. select a subset of the population to be allowed to `reproduce',
4. combine the selected subset of `parents' into `offspring',
5. introduce some `mutations' (changes) into the new offspring,
6. select the best P solutions from the sorted set of the parents and offspring.
7. repeat from step 2 for a given number of generations.

The evolutionary process was set to search for waveforms for a radar with a carrier of 10GHz, R_{mu}=100km, range resolution of 75m, compression ratio of 14 and velocity space of interest of ±500ms^{-1}. The chromosome structure used consists of N-1 genes, each gene being a PRI value ranging from 15μs (determined by being velocity unambiguous) to 667μs (determined by PRI being range unambiguous). Each gene was constrained to be rounded to a 1/10 μs value.

The evolutionary programme used a working population of 50 chromosomes and ran for 200 generations. The offspring were created by creating a copy of each parent chromosome in turn and then crossing over 5% of the genes with genes from another chromosome chosen at random from the parent population; which genes were swapped was chosen at random. The offspring chromosome was then altered through mutation where 20% of the gene values were perturbed by a small random value which was generated from a Gaussian distribution.

The Evolutionary algorithm was allowed to choose PRI values between 15μs (velocity unambiguous) and 667μs (range unambiguous). The EA was run 5 times and although on each run the solutions differed (suggesting the results are good local optima and not necessarily the global-optimum solution), they were of similar performance. The optimised PRI set analysed in this paper is 185.6, 335.8, 295.6, 102.6, 133.4, 26.7, 53.5, 362.7 and 666.7μs.

V. Results and Discussion

A. Method of Analysis

The waveforms have all been assessed based on the total dwell time required, the pattern of eclipsing in range and the overall ambiguity diagram. If DFT processing is applied as in (2), the ambiguity diagram may be formed as the product of the magnitude of the DFT profile in range for a zero velocity target, multiplied by the magnitude of the DFT profile in velocity for a zero range target. The resulting matrix may now be scaled by the maximum peak value in the matrix (i.e. the central peak) and provides the ambiguity diagram. For clarity, the range and velocity 'slices' are presented. All of the waveforms have been analysed over a range of 100km and a velocity space of ±500m/s.

B. Fibonacci Series

The timings based on the Fibonacci series result in a 9-pulse sequence of length 4370μs and a total duty ratio of 1.44%. Over the interval from zero to R_{mu}, this pulse sequence gives rise to a worst case eclipsing of just 2 pulses i.e. a loss factor of L_E = (9-2)/9 = 0.79 (-1.2dB), which is incurred at three specific ranges: 35.70km, 47.25km and 82.95km, see Figure 4 (top).

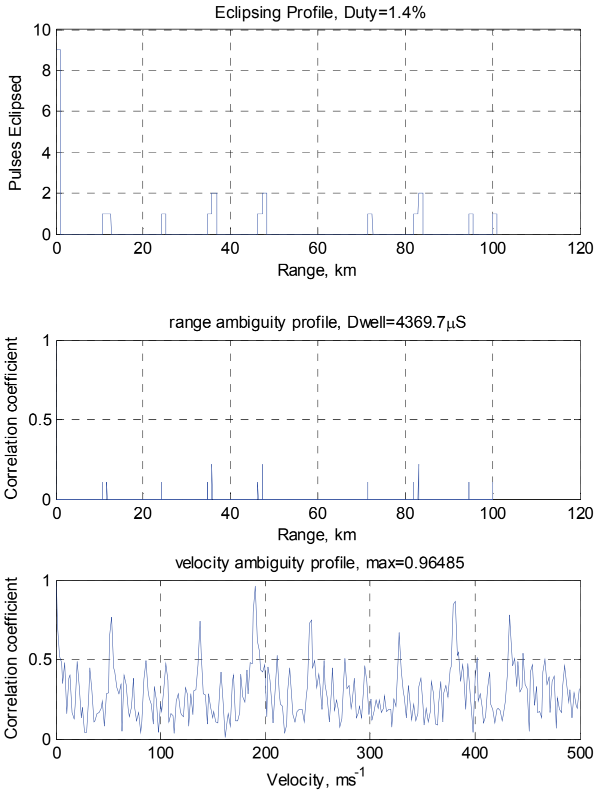

Figure 4: Results of Fibonacci Series Pulse Sequences
Top: Eclipsing Profile, Middle: Range Ambiguity Profile, Bottom: Velocity Ambiguity Profile

These ranges do not coincide with $t_d = t(k)$, although it may yet be possible to derive a pulse sequence resulting in a worst case eclipsing of just one pulse for a different initial choice of T_1 and T_2. The range ambiguity diagram, Figure 4 (middle) has peak side lobes of 2/9 whereas the velocity ambiguity diagram, Figure 4 (bottom) has multiple high level side lobes.

C. *Successive Doubling Sequence*

A 9-pulse sequence based on (12) results in a worst case eclipsing of just one pulse and has a total duty ratio of 0.47%. Longer codes derived on this basis will only lose one pulse, irrespective of the code length, giving a worst case eclipsing loss of $L_E = (N-1)/N$, where N is the number of pulses in the code. For this 9-pulse code, the eclipsing loss is 0.89 (-0.5dB) However, due to the PRI doubling, a large N results in a rapidly increasing CPI duration. The total duration of the code sequence investigated here is 13.4ms, which is long but not unreasonable. The range ambiguity diagram, Figure 5 (top) is very clear, having just a few side lobes of peak level 1/9. However, the velocity ambiguity diagram, Figure 5 (bottom) is poor.

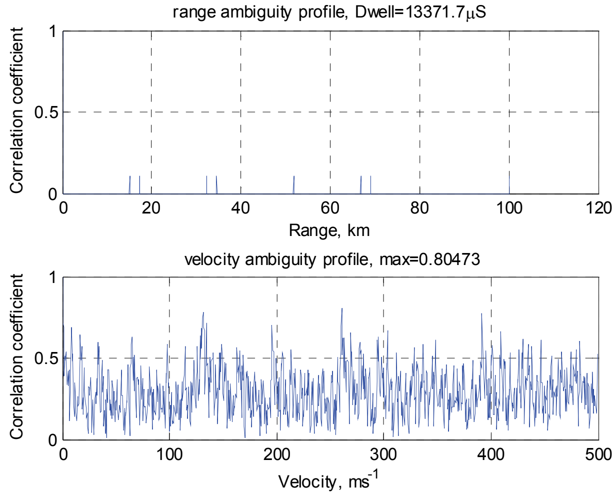

Figure 5: Results of Successive Doubling Series Pulse Sequences
Top: Range Ambiguity Profile, Bottom: Velocity Ambiguity Profile

D. *Costas Coded Sequence*

The 11 pulse sequence based on Costas coded sequences gives a dwell of 4.5ms and total duty ratio of 1.70% and incurs a worst case eclipsing loss (11-3)/11=0.73 (-1.4dB), within the range bracket of zero to 100km. Although this seems high, it only occurs at three specific ranges in the unambiguous range interval and the waveform possesses good ambiguity properties. The relatively high eclipsing losses affirm that (13) is *not* a good quality solution to the constraints of (4), irrespective of the transmitted pulse width. The range ambiguity diagram, Figure 6 (top) exhibits just five side lobes at a level of 1/9 for ranges below 60km, but side lobes of 2/9 are incurred beyond 60km. The velocity ambiguity diagram has modest side lobes at velocities below 200m/s, however, a response of unity amplitude is incurred close to 210m/s indicating that this is the maximum unambiguous velocity for this waveform.

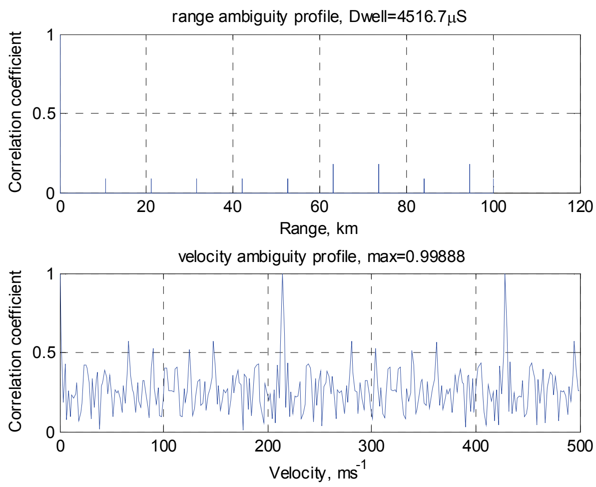

Figure 6: Results of Costas Coded Series Pulse Sequences
Top: Range Ambiguity Profile, Bottom: Velocity Ambiguity Profile

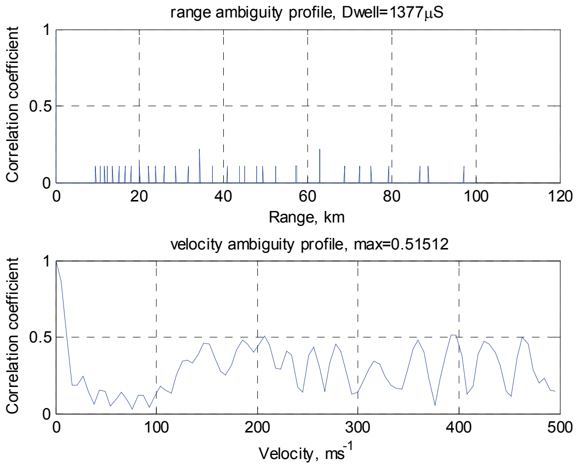

Figure 7: Results of Previous Medium PRF Pulse Sequences
Top: Range Ambiguity Profile, Bottom: Velocity Ambiguity Profile

E. Previous Medium PRF Solution

Using the parameters for the 9-pulse sequence results in a very short dwell of 1.38ms and a worst case eclipsing loss of (9-2)/9=0.78 (-1.1dB) for a total duty ratio of 4.6%. Whilst the losses are comparable to the Fibonacci series and slightly worse than the doubling sequence, the considerably higher duty ratio than the previous sequences has not caused a corresponding increase in the eclipsing losses. The range ambiguity diagram, Figure 7 (top), indicates many small side lobes with a peak value of 2/9. The velocity ambiguity diagram, Figure 7 (bottom), is quite clear of side lobes up to 100m/s but beyond this, they peak at an amplitude of 0.5.

F. Evolutionary Optimised Solution

The evolutionary process was used to identify a 9-pulse sequence that had a maximum eclipsing loss of (9-1)/9=0.89 (-0.5dB), see Figure 8 (top). The duty ratio of the waveform is 2.9%. The range ambiguity diagram, Figure 8 (middle), has a series of side lobes of amplitude = 1/9 and the velocity ambiguity diagram, Figure 8 (bottom), has side lobes up to an amplitude of 0.5.

VI. CONCLUSIONS

The pulse sequence derived by the Fibonacci timings suffer from modest eclipsing losses. The range ambiguity is reasonable but the velocity ambiguity performance is poor. Successive doubling has very good range eclipsing and range ambiguity performance but has velocity ambiguity problems and is a relatively long duration pulse sequence. Costas coding techniques give generally good eclipsing performance punctuated by a few poor ranges. The ambiguity performance is good in range out to 60km and, in velocity, would be very good up to 200m/s; it is not good in range or velocity over the full space of interest, though. It is possible that these shortcomings may be improved by scaling the timings appropriately.

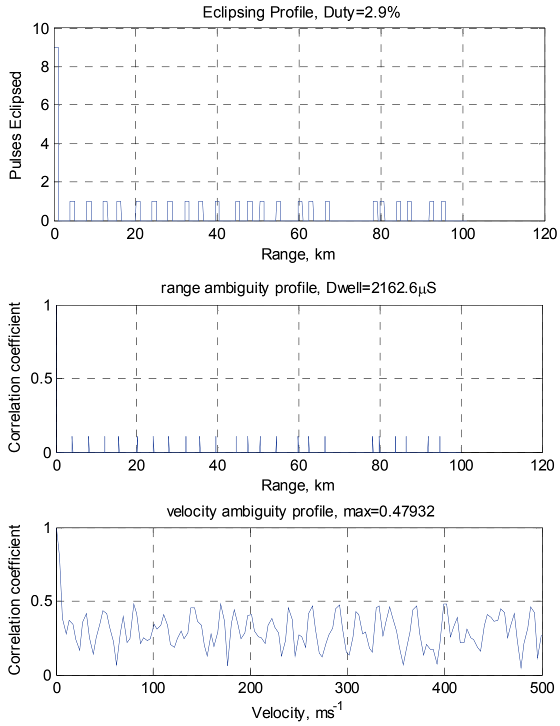

Figure 8: Results of Evolutionary Optimised Series Pulse Sequences
Top: Eclipsing Profile, Middle: Range Ambiguity Profile, Bottom: Velocity Ambiguity Profile

The timing sequence based on a near-optimum set of medium PRFs exhibits modest eclipsing losses. The ambiguity performance is poor in range but very good in velocity, particularly at low velocities; the schedule also only requires a very short dwell. The EA solution forms the best all-round waveform. It has several instances of low eclipsing losses which are more numerous and broader than those of the successive doubling sequence. Its range ambiguity performance is as good as successive doubling. The velocity ambiguity performance of the EA solution is the best of any of the coding methods trialled here over the entire space of interest. The velocity performance with only 9 pulses however is still poor compared to a full multi-dwell (*3 of 8*) medium PRF pulse Doppler approach, however, the EA solution described here only requires a fraction of the dwell time.

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