A Holographic Reconstruction Method for Circular Multistatic Subsurface Radar

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1. Introduction and motivation. Suburface Radar (SR) is a widely used technology for non destructive inspection and imaging. In recent years, SR has been successfully applied in novel applications such as breast cancer detection and wood inspection. The scan geometries in many of these applications are circular in order to better suit the shape of certain scan regions. This morphology also has an enhanced detection potential since the scan region can be irradiated from multiple views. Circular SR image formation presents a number of difficulties, such as the fact that the targets are in the near field region and the formation of non-linear signatures that complicate the visualization and detection of inclusions inside the scan region. A wide variety of approaches have been proposed to form circular SR images using both monostatic and bistatic scan protocols. Monostatic approaches are known to be more robust, whilst bistatic imaging methods exhibit a higher sensitivity. Nevertheless, the majority of these techniques are based on time-domain processing methods which yield images with artifacts and low Signal to Noise Ratios (SNR).

In this paper, a novel reconstruction technique for multistatic SR datasets acquired along circular scan geometries is proposed. This approach is based on the use of holography to properly migrate the responses from the targets in the scan area to their original spatial locations. Holographic techniques have previously been used for Circular Synthetic Aperture Radar (C-SAR) imaging, yielding accurate images [1,2]. However, C-SAR reconstruction methods cannot simply be applied for circular SR, since they are not designed to operate in scenarios were the targets are in the same scan plane as the antenna. Compared to current monostatic circular SR image formation approaches, the proposed technique generates images with a higher SNR and better focal quality. In this abstract, the theoretical formulation of the proposed approach will be described briefly and a series of preliminary results will be presented to evaluate the potential of the proposed approach.

2. Methodology. Let consider a circular scan geometry with a radius *R* and *N* scan locations. A waveform f(t) with a bandwidth *B* is radiated into the scan region from the scan location at (R, θ) and the reflections from the scan region are recorded at a series of *M* receiving locations. The collected response on the scan location at (R, φ) is given by:

$$s(t,\theta,\varphi) = \sum_{q=1}^{T} \sigma_q f\left(t - \frac{L_q(\theta) + L_q(\varphi)}{v}\right)$$
(1)

where $L_q(\theta) = \sqrt{R^2 + r_q^2 - 2Rr_q \cos(\theta - \phi_q)}$, (r_q, ϕ_q) are the polar coordinates of the q^{th} scatter and σ_q is its reflectivity. Under this signal model, monostatic measurements can also acquired in the case that $\theta = \varphi$. If we calculate the spherical phase function of the system, the resulting expression would be:

$$S(\omega,\theta,\varphi) = \sum_{q=1}^{T} \sigma_q F(\omega) \cdot exp\left(-jk\left(L_q(\theta) + L_q(\varphi)\right)\right)$$
(2)

where $k = \omega/v$ and $F(\omega)$ is the spectrum of f(t). To analyze the effects of the scan, the Fourier transform of (2) can be calculated with respect to α and β using a modification of the procedure described in [3]. Next, the phase components introduced by the scan geometry (given by the terms including R and π) are removed by performing a matched filtering process. The resulting spectrum is given by:

$$S(\omega, \alpha, \beta) = \sum_{q=1}^{T} \sigma_q \Gamma(\omega, \alpha, \beta) \cdot exp\left(-j(\Psi(\alpha) + \Psi(\beta))\right)$$
(3)

where $\Psi(\gamma) = \sqrt{k^2 r_q^2 - \gamma^2} + sin^{-1}(\gamma/k) + \gamma \phi_q$, α and β are the spatial frequency counterparts of θ and φ respectively, and γ is a placeholder variable and $\Gamma(\omega, \alpha, \beta)$ are the amplitude components of the signal in the (ω, α, β) frequency space. Next, the inverse Fourier transform of the compensated spectrum is calculated along yielding:

$$S_{c}(\omega,\theta,\varphi) = \sum_{q=1}^{T} \sigma_{q} \Theta(\omega) \cdot exp\left(-j2kr_{q}\left(\cos(\phi_{q} - (\theta + \varphi)/2)\cos((\varphi - \theta)/2)\right)\right).$$
(4)

To map the contents from $S_c(\omega, \theta, \varphi)$ to $S_c(\omega, \rho)$, where ρ are values related to the iso-doppler contours given by $\rho = (\theta + \varphi)/2$, a second matched filter is used to eliminate the effect of the term $cos((\varphi - \theta)/2)$. After this process is performed, the responses are added along the iso-doppler contours defined by ρ . To transfer the processed data to a rectangular frequency space $I(k_x, k_y)$, a mapping process given by $k_x = k \cdot cos(\rho)$ and $k_y = k \cdot sin(\rho)$ is performed. Finally, the inverse 2D Fourier transform of $I(k_x, k_y)$ is calculated yielding the final image i(x, y).

3. Results. The performance of the proposed method was assessed using a simulated dataset generated using the approach described in [4]. The irradiated waveform was a stepped frequency continuous wave with a center frequency of 3.5 GHz and a bandwidth of 6 GHz. The scan geometry had a radius of 20 cm and 72 scan elements that were used as transmitters and receivers. The simulation setup was modeled in such a way that it will mimic the materials found in a breast microwave radar imaging one of the most promising circular SR applications. In this particular setup, the scan region is formed by breast fatty tissue and a matching material was inserted between the antennas and the scan area. Five targets with an angular separation of 70 degrees at a radius of 4cm were inserted in the scan region. These targets have dielectric properties similar to breast tumors. The simulation setup and the reconstructed monostatic and multistatic image are shown in figure 1. The reflections of the dielectric interface were scale of the reconstructed images is in microwatts. Note that the multistatic image has a higher focal and lower noise levels compared to its monostatic counterpart. The performance of the proposed approach was also quantitatively assessed by calculating the SNR of the reconstructed images, their spatial accuracy, and their focal quality using entropy. These results are shown in Table I.



Figure 1. a) Simulation setup, b) Reconstructed monostatic dataset, c) Reconstructed multistatic dataset.

Table I. Ferformatice Metrics	Tab	le l	[. P	erform	ance	Metrics
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Dataset/Metric	SNR	Spatial error	Image entropy
Monostatic	0.98dB	(-5,3)mm	4.5 bits
Multistatic	6.81dB	(3,1.5)mm	2.5 bits

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