A Robust Fuzzy Autopilot Design using Multi-Criteria Optimization.

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Abstract

This paper details a Fuzzy - Feedback Linearisation controller applied to a non-linear missile. The design uses an evolutionary algorithm optimisation approach to a multiple model description of the airframe aerodynamics. A set of convex models is produced that map the vertex points in a high order parameter space (of the order of 16 variables). These are used to determine the membership function distribution within the outer loop control system by using a multi-objective evolutionary algorithm. This produces a design that meets objectives related to closed loop performance such as: steady state error, overshoot, settling and rising time. Two different ways for defining the objectives are used - one, convinient from an Engineering point of view, the other suitable for the Optimization world. The evolutionary algorithm uses non-dominated sorting for forming a Pareto front of possible solutions. The concept of crisp preferability has given useful information to the optimizer, which is faced with sampling a very large tradeoff surface. While when applying penalty preferability (handling objectives as constraints) the searching process for forming the Pareto front is relieved and is probable that all the non-dominated solutions are acceptable, so the engineering design requirements are satisfied.

1 Introduction

The problem considered here is that of tracking a trajectory in the presence of noise and uncertainty. Many nonlinear analysis problems of engineering interest can be reduced to such a problem. Since the real system is not exactly the one used for the design, and since it is also subject to noise, the system will not follow the intended trajectory. Then the question of interest becomes: will the real trajectory, under the worst conditions possible, remain close enough to the nominal one. This could be defined as a robust trajectory tracking problem. Here, this kind of problem is addressed for a highly non-linear missile when the design of an autopilot is taken into account. Although such systems are well defined in terms of their dynamic behaviour, they have large uncertainty in their parameters and can cover large ranges of altitude and speed. By demanding small changes in system outputs, it is possible to exhibit the non-linear behaviour of the system, then a robust non-linear control technique is required to achieve good performance.

The aim of this paper is to track the missile lateral acceleration demand in the presence of uncertainties introduced through the aerodynamic coefficients. The g demands are considered for both pitch and yaw planes, using the missile rudder and elevator as control surfaces hence yielding a system with 2 inputs and 2 controlled outputs.

It has been shown previously [1] that by applying feedback linearization the desired tracking performance can be obtained by assuming an exact knowledge of aerodynamic coefficients and missile configuration parameters (i.e., reference area, Mach number, mass, moment of inertia) in the entire flight envelope. In practice however, this assumption is not valid and also, if there are either parameter variations or external disturbances, feedback-linearisation can no longer guarantee the desired performance (neither is robustness guaranteed).

Conversely fuzzy logic theory is useful when dealing with vague and imprecise information such as uncertain measurement values, parameter variations and noise [2]. In previous research [3] a combination of an input/output

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Figure 1: Highly nonlinear maneuvre

linearisation technique (nonlinear control law) and a fuzzy logic trajectory controller have been considered. The design uses an evolutionary algorithm optimisation approach to a multiple model description of the airframe aerodynamics. This is used to determine the membership function distribution within the outer loop control system by using a multi-objective EA that meets objectives related to closed loop performance such as: rising and settling time, steady state error, and overshoot.

2 HORTON Missile model

The missile model used in this study derives from a non-linear model produced by Horton of Matra-British Aerospace [4]. It describes a 5 DOF model in parametric format with severe cross-coupling and non-linear behaviour. This study has considered the reduced problem of a 4 DOF controller for the pitch and yaw planes without roll coupling. The angular and translational equations of motion of the missile airframe are given by:

$$\begin{aligned} \dot{q} &= \frac{1}{2} I_{yz}^{-1} \rho V_o S d \left(\frac{1}{2} d C_{mq} q + C_{mw} w + V_o C_{m\eta} \eta\right) \\ \dot{w} &= \frac{1}{2m} \rho V_o S (C_{zw} w + V_o C_{z\eta} \eta) + Uq \qquad (1) \\ \dot{r} &= \frac{1}{2} I_{yz}^{-1} \rho V_o S d \left(\frac{1}{2} d C_{nr} r + C_{nv} v + V_o C_{n\zeta} \zeta\right) \\ \dot{v} &= \frac{1}{2m} \rho V_o S (C_{yv} v + V_o C_{y\zeta} \zeta) - Ur \qquad (2) \end{aligned}$$

where the axes(x, y, z), rates(r, q) and velocities (v, w) are defined in (Figure 2).

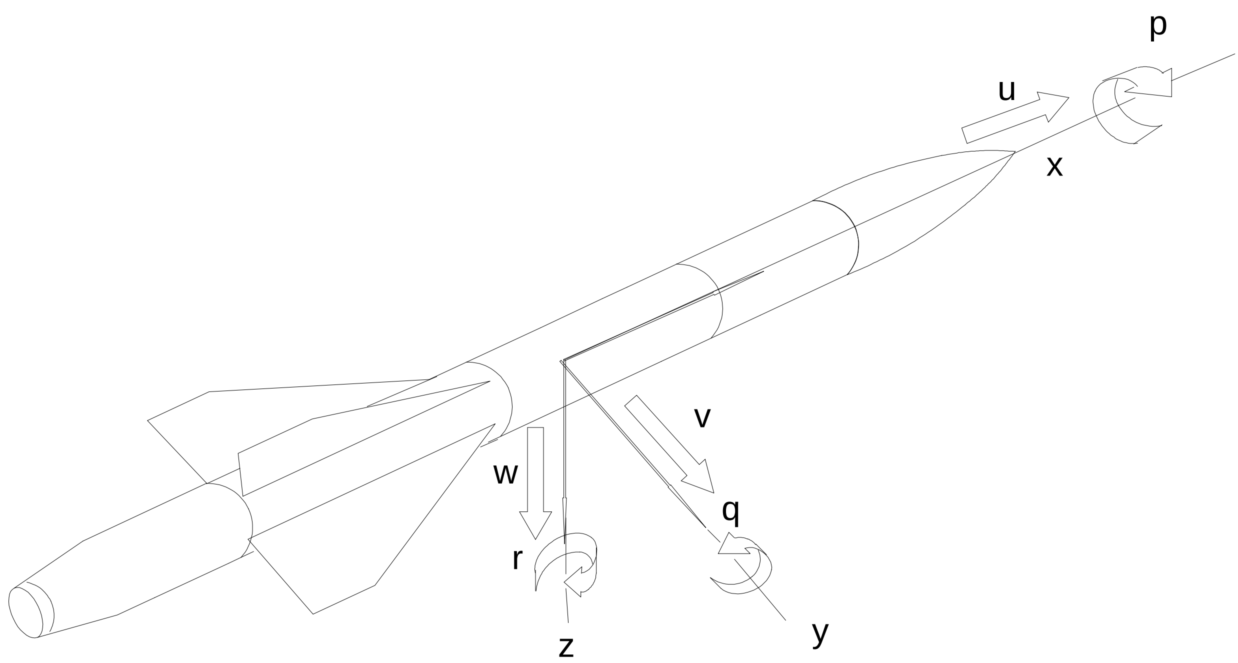

Figure 2: Airframe axes

Equations (1,2) describe the dynamics of the body rates and velocities under the influence of external forces (e.g. C_{zw}) and moments (e.g. C_{mq}), acting on the frame. These forces and moments, derived from wind tunnel measurements, are non-linear functions of Mach number longitudinal and lateral velocities, control surface deflection, aerodynamic roll angle and body rates.

The aerodynamic coefficients: C_{yv}, $C_{y\zeta}$, C_{nr}, C_{nv} and $C_{n\zeta}$ are presented by polynomials shown in Table 1 and Table 2. These polynomials are fitted to the set of curves taken from look-up tables for different flight conditions (roll angle 0° and 45°).The tables cover the aerodynamic coefficients in supersonic range for different roll angles 0° and 45°. They are a set of curves in the plane of total incidence σ in $[rads]$ and Mach number M. In these tables the c_{yv} polynomials present the *normal force* curves, the x_{cp} present the *centre of pressure* curves, c_{yz} present the *rudder and elevator control forces* curves, and finally the c_{nr} present the *damping yawing and pitching moments* curves which are reasonably proportional to body rates.

The carpet plot for these functions is shown in figure 3, plotted as a function of incidence and roll angle for different mach numbers (M), with figure 4 plotted as a function of mach number and roll angle for different incidence angles (σ).

Normal force	$c_{yv_0} = -25 + 1.0M - 60\sigma$
Control surfaces	$c_{y\zeta_0} = -10 - 1.6M + 2.0\sigma$
Centre of pressure	$x_{cp_0} = 1.3 + 0.1M + 0.2\sigma$
Damping moment	$c_{nr} = -500 - 30M + 200\sigma$

Table 1: Roll angle = 0°

Normal force	$c_{yv_0} = -26 + 1.5M - 30\sigma$
Control surfaces	$c_{y\zeta_0} = -10 - 1.4M + 1.5\sigma$
Centre of pressure	$x_{cp_0} = 1.3 + 0.1M + 0.3\sigma$
Damping moment	$c_{nr} = -500 - 30M + 200\sigma$

Table 2: Roll angle = 45°

The description of the model is obtained from data supplied by Matra-BAE and detailed in the Horton report [5]. As both horizontal and vertical lateral motion is symmetric in format, both will be dealt with together, taking into account the appropriate sign changes in derivatives for each lateral direction.

The control of the missile will be accomplished in this paper by controlling lateral velocity. The dynamic equation for lateral velocity can be derived as following:

$$\begin{aligned}
\dot{v} &= V^o(C_{yv}v + V_oC_{y\zeta}\zeta) - Ur \\
&= V^o[(C_{yv_0} + C_{yv_M}M + C_{yv_\sigma} \mid \sigma \mid)v \\
&\quad + V_o(C_{y\zeta_0} + C_{y\zeta_M}M + C_{y\zeta_\sigma} \mid \sigma \mid)\zeta] - Ur \\
&= V^o[(\bar{C}_{yv_0}v + \bar{C}_{yv_\sigma} \mid v \mid v \\
&\quad + V_o\bar{C}_{y\zeta_0}\zeta + V_o\bar{C}_{y\zeta_\sigma} \mid v \mid \zeta] - Ur
\end{aligned} \tag{3}$$

where the Mach number M, and the total velocity V_o are slowly varying and where:

$$\begin{aligned}
\mid \sigma \mid &= \frac{\mid v \mid}{V_o}\frac{180}{\pi} \\
M &= \frac{V_o}{SoS} \\
V^o &= \frac{1}{2m}\rho V_o S \\
\bar{C}_{yv_0} &= C_{yv_0} + C_{yv_M}\frac{V_o}{SoS} \\
\bar{C}_{yv_\sigma} &= C_{yv_\sigma}\frac{180}{V_o\pi} \\
\bar{C}_{y\zeta_0} &= C_{y\zeta_0} + C_{y\zeta_M}\frac{V_o}{SoS} \\
\bar{C}_{y\zeta_\sigma} &= C_{y\zeta_\sigma}\frac{180}{V_o\pi}
\end{aligned} \tag{4}$$

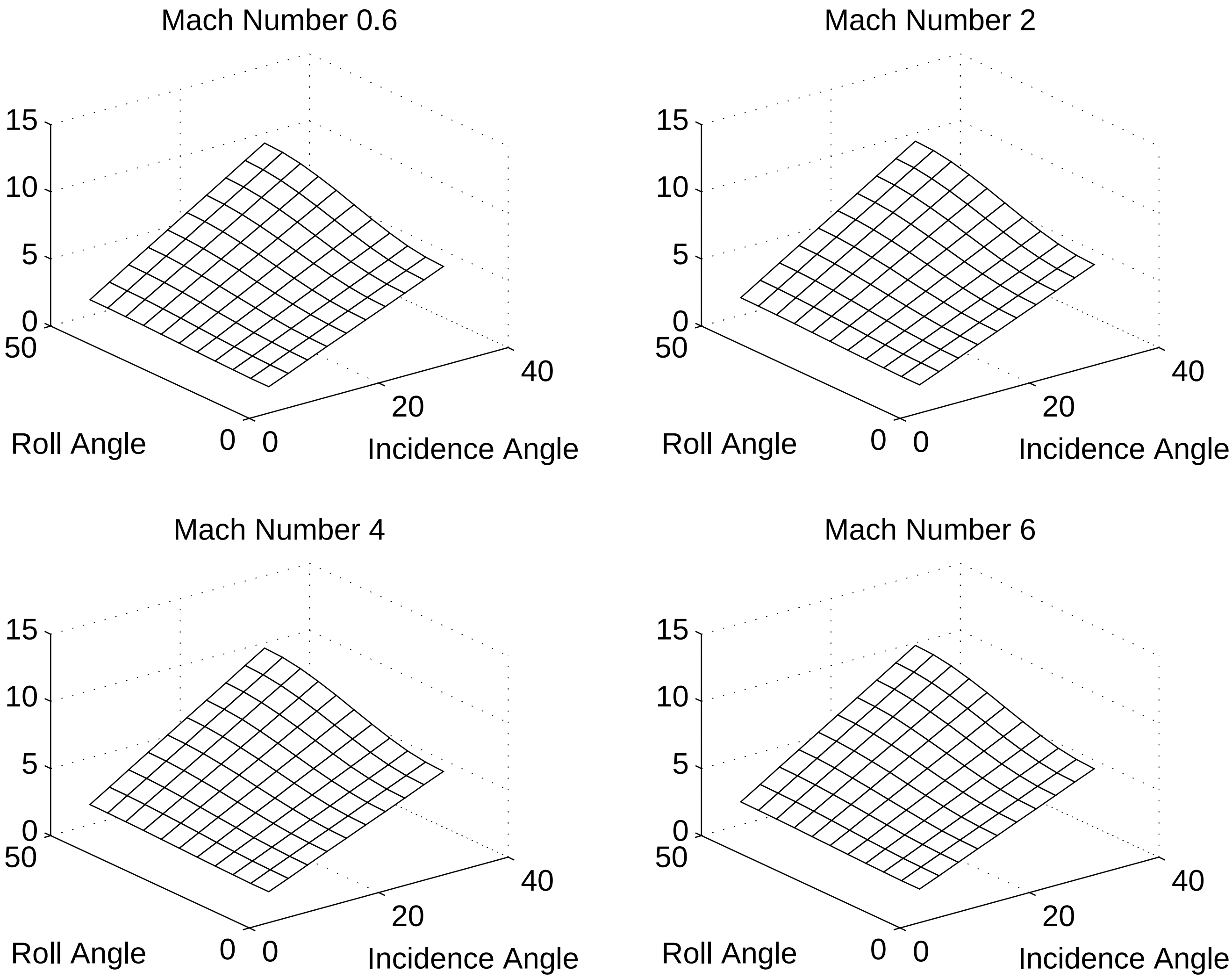

Figure 3: Centre of Pressure for different Mach Numbers

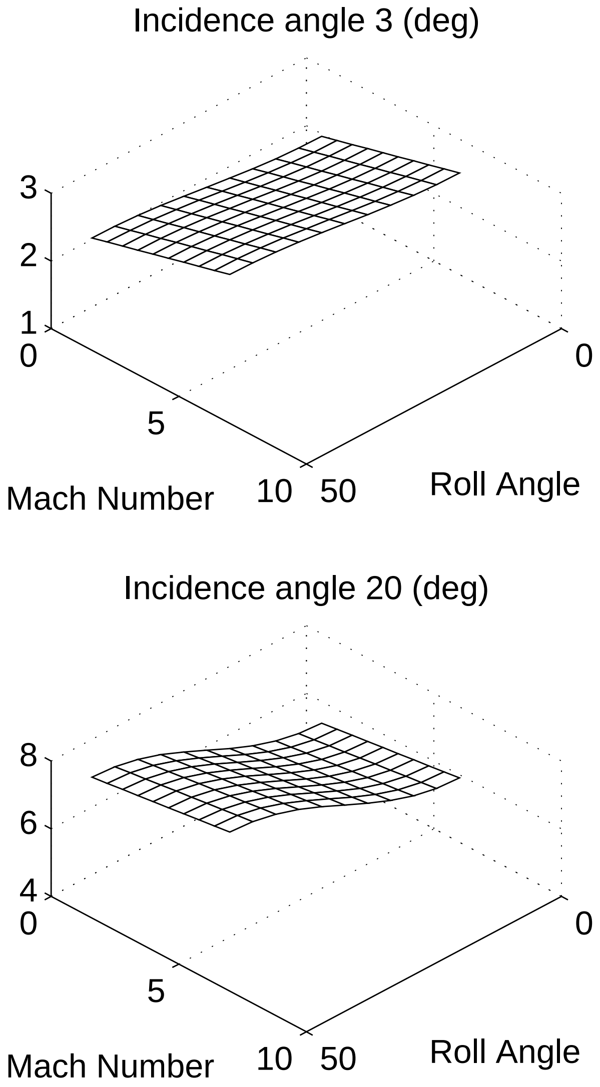

Figure 4: Centre of Pressure for different Incidence Angles

The state-space form of the non-linear system of the missile is written in a matrix form:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h \\ &= \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} \end{aligned} \tag{5}$$

For the selected outputs (lateral velocities) an approximate input-output linearisation has been applied in our previous work [1]. A combination of neglecting sufficiently small terms during the differentiation process and proposing an output that is an approximation of the desired one has been used which has resulted in a linear equivalent system with total relative degree equal with the order of the system. This means there is no part of the system dynamics which is rendered "unobservable" in the approximate input-output linearization. Since there is no internal (zero) dynamics the stability of the linearized system can be guaranteed and the tracking problem is solved [6] .

The effect of neglecting small terms (the side-slip force acting on the control surfaces) in the g vector field is to eliminate a non-linear zero in the system within the model description, and which is not taken into account in the non-linear control design. It has been shown in [7] that provided the side-slip force is not too great this will not affect the performance of the control design in a significant manner.

The required static state feedback for decoupled closed loop input/output behaviour is given by [6] as:

$$u = E^{-1}\left\{ v - \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \right\} \tag{6}$$

where E^{-1} is the decoupling matrix of the system and it is nonsingular.

After applying the feedback linearisation technique the linearised closed loop system can be written as:

$$\ddot{y}_i = v_i \tag{7}$$

where v is the new linearised system input.

It has been shown [1] that the desired tracking performance for lateral acceleration can be obtained by assuming an exact knowledge of aerodynamic coefficients and missile configuration parameters (i.e., reference area, Mach number, mass, moment of inertia). In practice however, this assumption is not valid and also, if there are parameter variations or external disturbances, feedback-linearisation can no longer guarantee the desired performance or neither is robustness guaranteed. For these reasons, a combination of an input/output linearisation technique (nonlinear control law) and a fuzzy logic controller(trajectory controller) have been chosen to be considered here.

3 Multi-modelling: Sensitivity Analysis

The variations in aerodynamic coefficients have introduced parametric uncertainties into the non-linear system. A large excursion on perturbations within the whole range of aerodynamic roll angles 0° and 45° have been examined and perturbations on each of the aerodynamic coefficients ($c_{yz}, c_{yv}, x_{cp}, c_{nr}$) have been introduced into the system in a large variety of percentage deviation from nominal values.

Based up on simulations it has been found that some coefficients can be allowed larger percentage variation from the nominal case than others. Within the system we are able to tolerate ±50% uncertainty in c_{yz}, c_{yv}, c_{nr} before it goes unstable.

Also it has been found that the centre of pressure x_{cp} and the control surfaces c_{yz} polynomials have most significant effect on the close loop performance (the system is very sensitive to small changes) while the damping moment contribution in c_{nr} is small and the system is almost insensitive so can be simplified to be independent aerodynamic roll angle.

The sign of x_{cp} can tell us whether the system is stable or not. When the σ term of x_{cp} is varied to around +50% change we get an unstable system.

In a real flight scenario, for every instance of this missile type, the aerodynamical functions may deviate from their nominal values, taken in wind tunnel measurements. In order to explore the complexity of the problem we have assesed the open and cloosed loop system for different autopilot demands by computing the lateral acceleration and sideslip velocity errors. For simplicity we have studied the single plane (lateral or vertical motion) when roll angle is 0°.

Figure 6 shows that demanding 5g ($50[m/sec^2]$) lateral acceleration and using the fixed gains trajectory controller, feedback linearisation is not able to give robust performance, neither is the desired tracking achieved when perturbations on the aerodynamical coefficients exist, which is the case in reality.

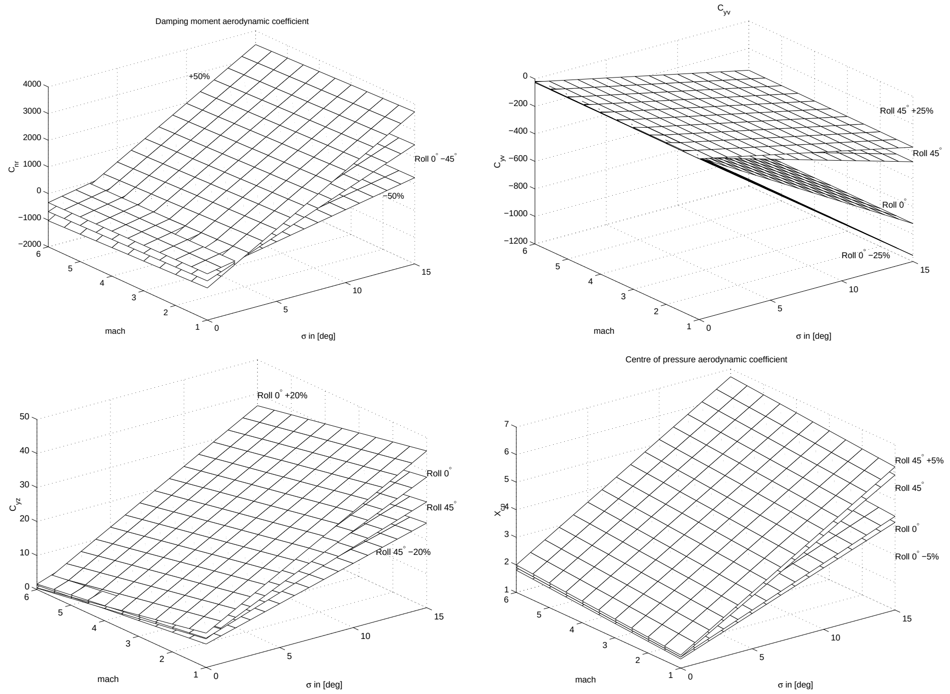

Figure 5: Aerodynamic coeffs ranges

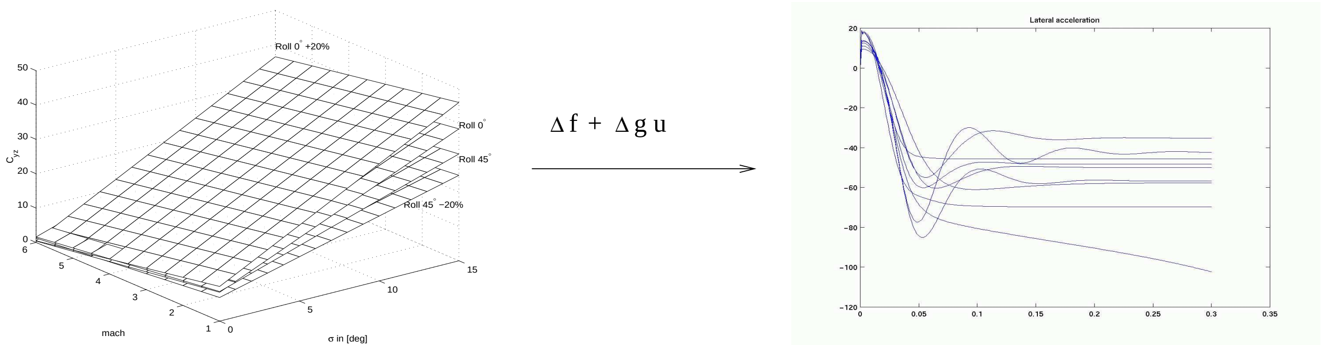

Figure 6: Uncertainties through aerodynamic coefs

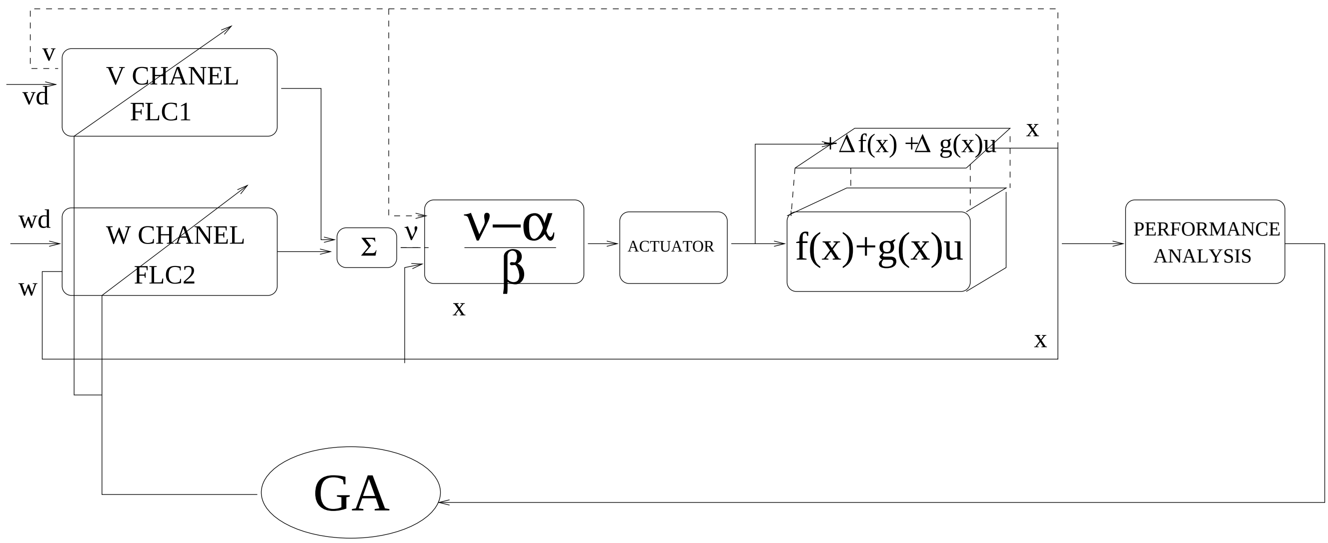

Figure 7: Trajectory control design

4 Fuzzy trajectory controller

The autopilot design shown in figure 7 consists of the missile dynamics presented by

$$\begin{aligned} \dot{x} &= f(x) + \triangle f(x) + (g(x) + \triangle g(x))u \\ y &= h(x) \end{aligned} \tag{8}$$

which presents the multi-modeling frame. A fast $250[rads/sec]$ second order linear actuator is included within the missile dynamics. Fin angles and fin rates are states of the system. The non-linear control law $\frac{\eta-\alpha}{\beta}$ derived by the feedback linearization technique, which decouples the system. Two fuzzy logic trajectory controllers are used in the outer loop for the lateral v, and vertical w channels respectively. An optimization procedure is used to tune the membership functions and the rules of the Fuzzy Logic Controller. The fixed gains used in the design of the nominal model correspond to natural frequency $w_n = 50(rad/sec)$ and damping factor $\zeta = 0.7$ of the closed loop system.

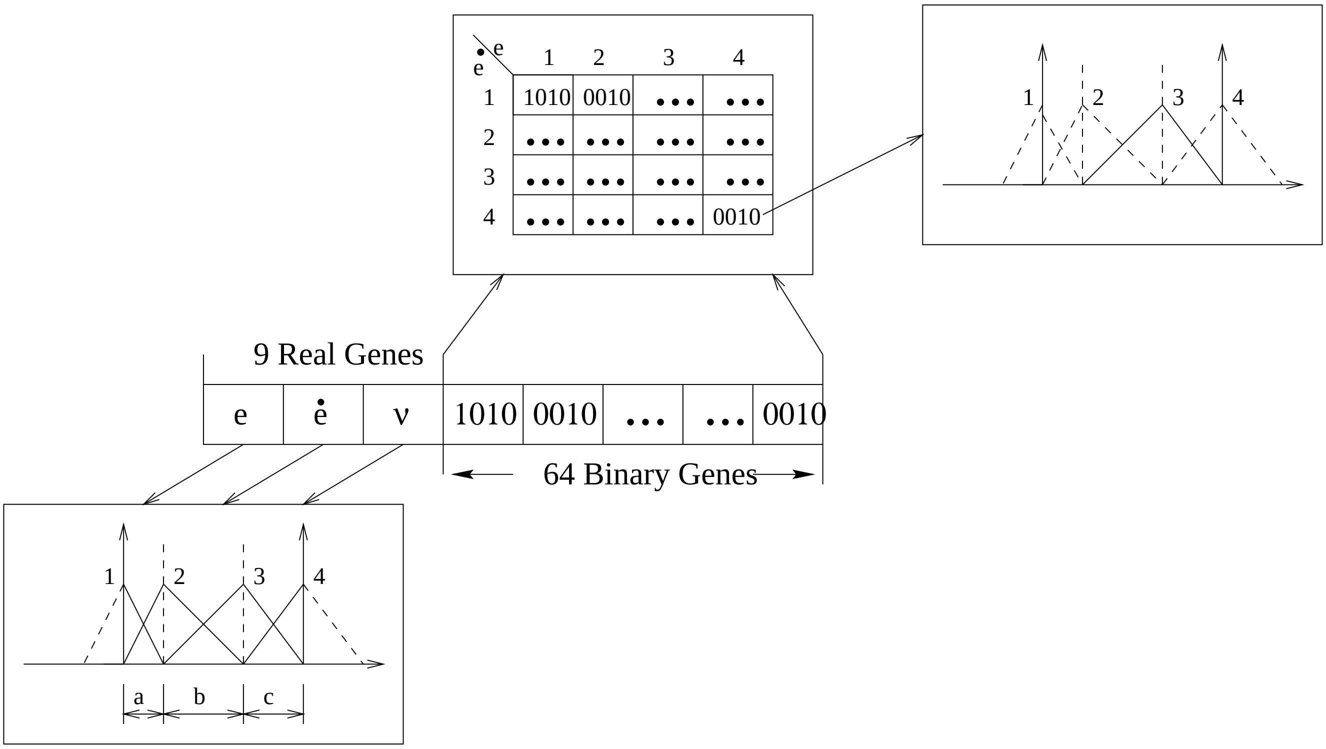

Figure 8: FLC chromosome structure

The trajectory controller has been designed, based on a fuzzy inference engine, as a two input - one output system with four membership functions for each variable (see figure 8). The membership functions' positions and the rules are generated using an evolutionary algorithm.

It has been found that for a range of $\pm 25\%$ change, the aerodynamic coefficient C_{yv} can vary before the side-slip velocity exceeds 10% steady state error within the feedback linearised loop. For similar performance, C_{yz} can vary by $\pm 15\%$, and the most sensitive coefficient, X_{cp}, can vary by $\pm 1.5\%$.

5 Evolutionary Algorithm Structure

The proposed framework maintains a population of fuzzy rule sets with their membership functions and uses the evolutionary algorithm to automatically derive the resulting fuzzy knowledge base.

A hybrid real valued/binary chromosome has been used to define each individual fuzzy system. The real valued parameters are defined as being the $[\delta a\, \delta b\, \delta c]$ values shown in figure 8. The binary component encodes the set of rules used in the system. Each rule is either on or off (0/1) and corresponds to the form:

$$\text{if } A_i \text{ AND } B_j \text{ then } O_k \tag{9}$$

where A_i denotes membership function i of input A, B_j denotes membership function j of input b, and O_k denotes membership function k of the output O. This process allows a full set of rules to be developed for the fuzzy system, but maintains a fixed length chromosome. The four membership function structure leads to a chromosome with 9 real valued genes and 64 binary genes. The fuzzy system used product for the member function 'AND'. The 'OR' function was not required as the rules were all expressed as 'AND' terms. The implication method was to chose the minimum value and crop the output member functions. The aggregation method was to choose the maximum values of the set of member functions. A centroid approach was used to defuzzify the output.

The evolutionary algorithm[8] follows the usual format of ranking, selection, crossover, mutation and evaluation but with the real and binary parts of the chromosomes being processed separately. The same number of offspring are generated as parents and a total replacement policy is used. This helps slow convergence and helps to reduce the effects of the noisy objective functions. A multi-objective approach was used to identify good solutions. A method known as non-dominated ranking was used in the evolutionary algorithm to allow the multi-objective problem to be handled easily. A detailed description of the non-dominated ranking process may be found in [9], and is based on several layers of classifications of the individuals. Before selection is performed the population is ranked on the basis of nondomination: all nondominated individuals are classified into one category (with a dummy fitness value, which is proportional to the population size, to provide an equal reproductive potential for these individuals). To maintian the diversity of the population, these classified individuals are shared using their dummy fitness values. Then this group of classified individuals is ignored and another layer of nondominated individuals is considered. The process continues until all individuals in the population are classified.

Stochastic universal sampling is used to select good individuals from the population for breeding. Since individuals in the first front have the maximum fitness value, they always get more copies than the rest of the population. This allows the algorithm to search for nondominated regions, and results in convergence of the population toward such regions. Sharing, by its part, helps to distribute the solutions over this region. The main strengths of this technique is that it can handle any number of objectives independently.

6 Optimistic Reference point approach

Depending on how we consider those objectives will affect the Evolutionary algorithm behaviour in terms of convergence and searching through feasible regions for acceptable solutions. Previously [3] we have used the surrogate additive function which transfers the vectorised multi-objective problem into a scalar optimization problem.

One way to explore this problem is to define closed loop performance criterias as four objectives by using the reference point approach [10] Other researchers [11],[12] have applied similar ideas by using goal attainment method to a gas turbine engine model.

Having the objectives are expressed in a direct form is very attractive, its practical usefulness depends on the ability to search and optimize cost surfaces in a class much broader than usual. We demonstrate the need of preference articulation in cases where many and highly competing objectives lead to a non-dominated set too large for a finite population to sample effectively. While if those objectives are defined as inequality constraints (penalties) this will reduce the nondominated set at the final population and is guaranteed that all the solutions on the Pareto front are acceptable for the designer/engineer. Because the EA is searching in a larger feasible area for good solutions, in the first case it is shown that only a subset of the nondominated set is of practical relevance, which further substantiates the need for further interaction with the Decision Maker (i.e. selective process at the end of the optimization).

The tradeoff information generated by the evolutionary algorithm can contribute to a better understanding of the complexity of the problem. As we have used non-dominated sorting for forming the Pareto front, the concept of crisp preferability can be seen to provide useful information to the optimizer which would otherwise be faced with sampling a very large tradeoff surface (if all the closed loop performance criterias are defined in direct form). While when applying penalty preferability, handling objectives as constraints can relieve the searching process for forming the Pareto front and it is probable that all the solutions are acceptable, so the engineering design requirements are satisfied.

Preference articulation [13] can be given by assigning weight coefficients, priorities, or goal values which indicate desired levels of performance in each objective dimension. The way goals are interpreted may vary. The goals may represent minimum levels of performance to be attained, Utopian performance levels to be approximated, or ideal performance levels to be matched as closely as possible. Goals are usually easier to set than weights and priorities, because they relate more closely to the final solution of the problem.

In the optimistic approach the decision maker initially specifies optimistic objective function values (not achievable simultaneously) as the desired values. A solution is found by minimizing the underachievements of the objective function values with respect to the specified desired values. This solution is presented to the decision maker who is asked to revise his/her expectations. The optimistic approach can be viewed as the special case of the reference point approach in which all reference values consistently exceed the objective function values at all the intermediate solutions.

Generally the objective criteria are incomparable and the numerical values may differ considerably. A procedure for normalisation must be used to convert the criteria $y_j(\mathbf{x})$ into a dimensionless function $\eta_j(\mathbf{x})$ for which usually $\eta_j(\mathbf{x}) \in [\mathbf{0}, \mathbf{1}]$.

The optimistic reference point approach [10],[14] known also as using a function of losses represents the losses from the ideal values $y_j^\star$ for the objectives:

$$\eta_j(x) = \frac{y_j^\star - y_j(\mathbf{x})}{y_j^\star}, j \in [[1, \ldots, m. \tag{10}$$

If the ideal values $y_j^\star$ are very small numbers or $y_j^\star \longrightarrow$, an alternative form can be used

$$\eta_j(x) = \frac{y_j^\star - y_j(\mathbf{x})}{y_{jmax} - y_{jmin}}, j \in [[1, \ldots, m. \tag{11}$$

Where y_{jmax} and y_{jmin} are respectively the maximum and minimum values of the criterion $y_j(x)$ in $x \in X_0$ subject to 11. The surrogate function of losses should be minimized such that:

$$minF(y(\mathbf{x})) \equiv \mathbf{minF}(\mathbf{x}) = \mathbf{min} \sum_{\mathbf{j=1}}^{\mathbf{m}} [\eta_{\mathbf{j}}(\mathbf{x})]^{\mathbf{2}} \tag{12}$$

This approach is called *optimistic*, because the most desired values $y_j^\star$ for the objectives in equation 10 are chosen. This is applied to all four closed loop performance criterias: rising time, steady state error, overshoot and settling time.

The closed loop performance criteria are choosen as following:

- Acceleration steady state error:

$$\eta_j(x) = \frac{Er_j^\star - Er_j(\mathbf{x})}{Er_{jmax} - Er_{jmin}}, j \in [1, \ldots, m]. \tag{13}$$

- Overshoot:

$$\eta_j(x) = \frac{Os_j^\star - Os_j(\mathbf{x})}{Os_{jmax} - Os_{jmin}}, j \in [1, \ldots, m]. \tag{14}$$

- Rise time:

$$\eta_j(x) = \frac{Tr_j^\star - Tr_j(\mathbf{x})}{Tr_{jmax} - Tr_{jmin}}, j \in [1, \ldots, m]. \tag{15}$$

- Settling time:

$$\eta_j(x) = \frac{Ts_j^\star - Ts_j(\mathbf{x})}{Ts_{jmax} - Ts_{jmin}}, j \in [1, \ldots, m]. \tag{16}$$

Table 3 shows the reference points used in the objective calculations.

Reference points	Steady State Error	Settling time	Rising time	Overshoot
Ideal point	$Er_j^\star = 0.0[\%]$	$Ts_j^\star = 0.15[sec]$	$Tr_j^\star = 0.08[sec]$	$Os_j^\star = 4.5[\%]$
Maximum	$Er_{jmax} = 2.0[\%]$	Ts_{jmax}=0.25$[sec]$	$Tr_{jmax} = 0.14[sec]$	$Os_{jmax} = 25.0[\%]$
Minimum	$Er_{jmin} = 0.0[\%]$	$Ts_{jmin} = 0.1[sec]$	$Tr_{jmin} = 0.07[sec]$	$Os_{jmin} = 2.0[\%]$

Table 3: Closed loop performance criterias

7 Results

The fuzzy surface has been developed with the model exercising the nominal aerodynamic coefficients. Figure 9(a) shows the full fuzzy surface of the trajectory controller generated by the evolutionary algorithm. Figure 9(b) shows the percentage usage of the different regions of the full control surface. It is clear that only a small proportion is actually used and therfore 'tuned' by the evolutionary algorithm. Figure 9(c) shows the section of the surface that is used, with a contour plot of the actual usage shown below it. Figure 9(d) shows the sideslip velocity response for the nominal case and two cases with extreem coefficient values (detailed below). The rise time is 0.07 seconds, overshoot 0.18%, settling time 0.2 seconds, and 0.038% settling error for the nominal case. The most fired rule (70%) in this solution is when both error and the derivative of the error are zero, which is the steady state area of the response.

The fuzzy logic controller has been tuned for the nominal case of the aerodynamical coefficients, demand $2.57[m/sec]$ corresponding to 1g pull lateral acceleration, and tested for parameter variations within the ranges specified in section 4. Two particular combinations of variation have been used:

1. $[\ C_{yv_{min}} \quad C_{yz_{min}} \quad C_{nr_{max}} \quad X_{cp_{max}}\ ]$
2. $[\ C_{yv_{max}} \quad C_{yz_{min}} \quad C_{nr_{max}} \quad X_{cp_{min}}\ ]$

Robust performance to these errors within 5.4% relative steady state error has been achieved.

Figure 10 shows the surface section used and the corresponding side slip velocity responses for two other solutions in the nondominated set. The upper solution has an unsatisfactory overshoot (37%) and steady state error (32%), but has a fast rise time (0.06 sec) and good settling time (0.2 sec). The lower solution has an unsatisfactory rise time (0.17 sec) and steady state error (63%), but the settling time is good (0.2 sec) and it exhibits no overshoot.

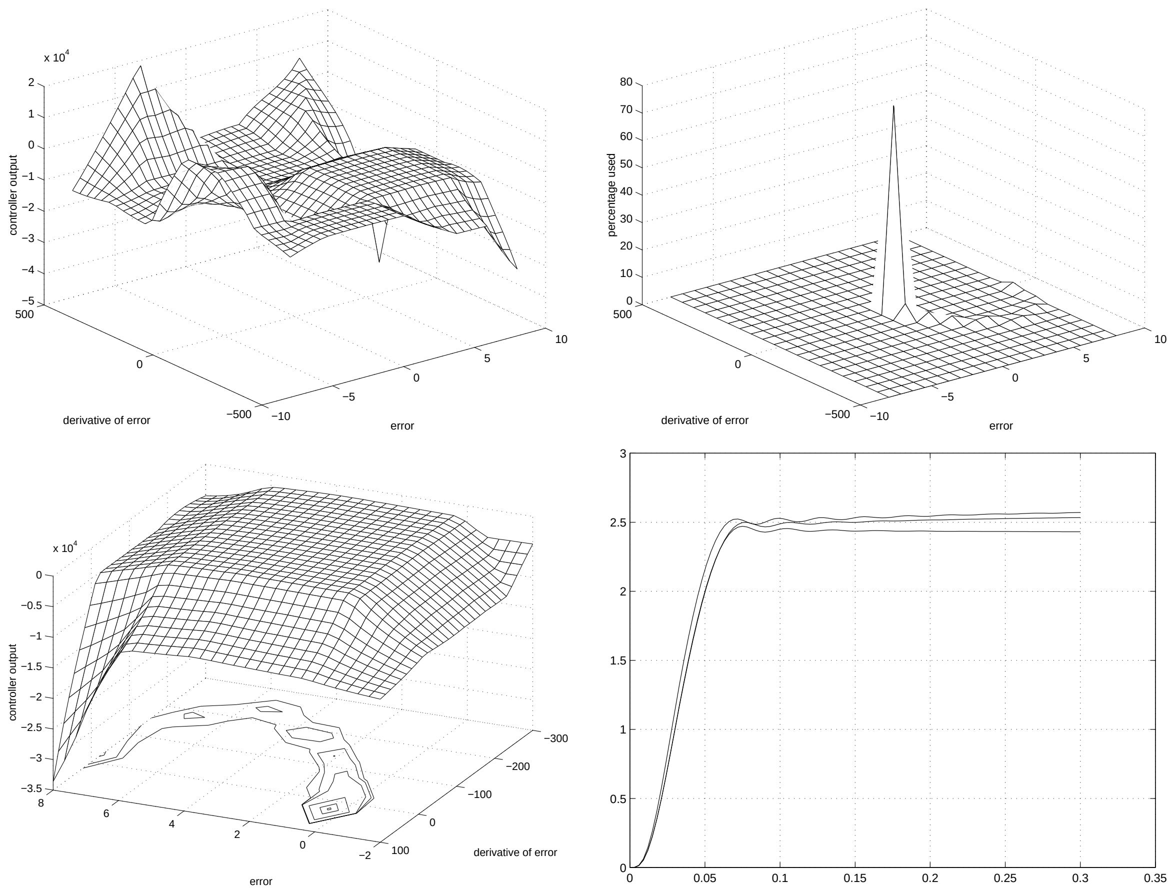

Figure 9: a)FLC surface b)usage surface c)FLC used area d)sideslip velocity

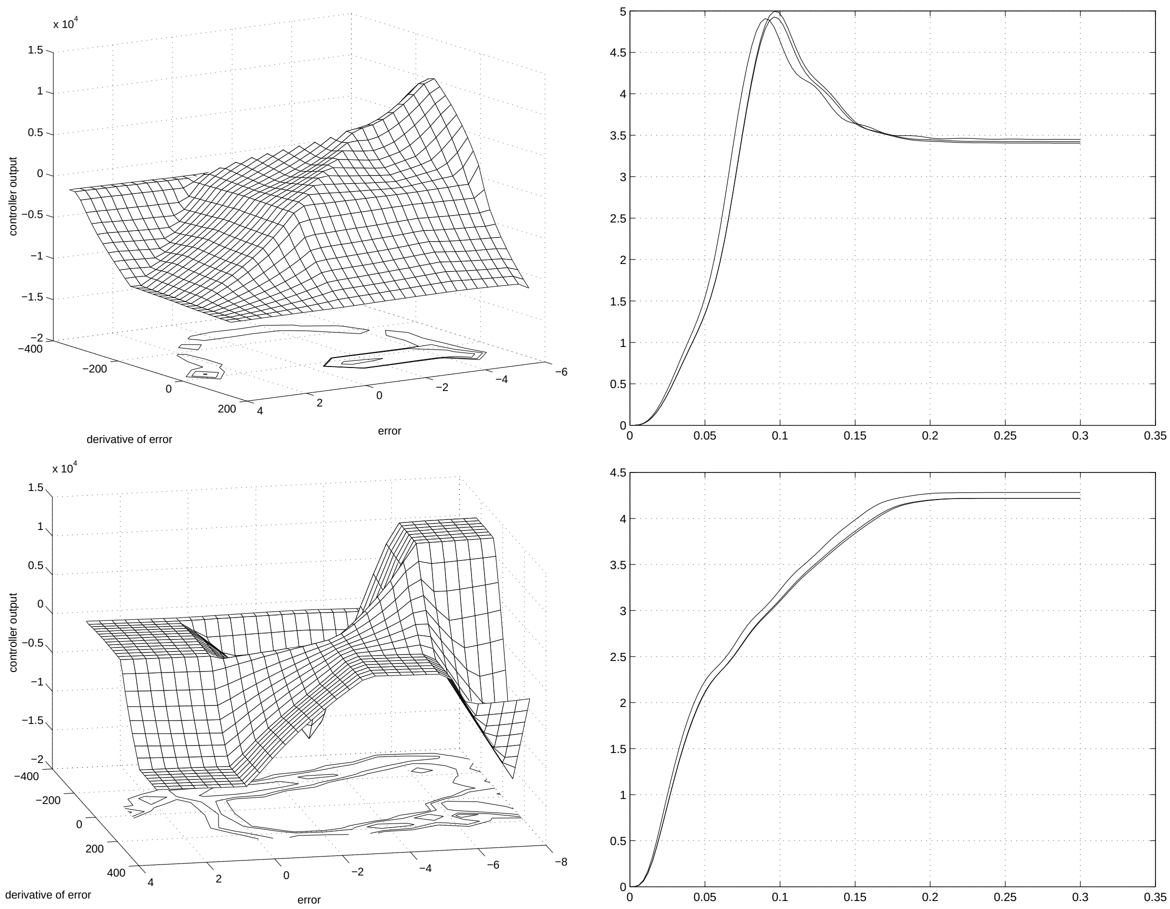

Figure 10: Two alternative Pareto solutions

8 Conclusions

We have evaluated the robustness of feedback linearisation on the significant parametric uncertainty introduced into the system through the aerodynamic coefficients. We have proposed a fuzzy outer loop to improve the robustness. We have shown that the evolutionary algorithms can produce a good set of results that populate the Pareto solution set, allowing the system designer the flexibility of trading one solution against others to achieve a desired performance.

As figure 9 demonstrates, by monitoring the usage of the controller surface, combined with the fuzzy control approach, an insigt can be gained into the operation of robust controllers that are created using the evolutionary process.

9 Acknowledgements

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A Nomenclature

x	roll axis
y	pitch axis
z	yaw axis
q	Pitch rate
r	Yaw rate
U	Velocity along the roll axis
v	Velocity along the pitch axis
w	Velocity along the yaw axis
V_o	Total Velocity
η	Elevator angle
ζ	Rudder angle
I_{yz}	Inertia
m	Mass of the airframe
d	Missile diameter
S	Wing chord
ρ	Air density
x_{cg}	Centre of Gravity
x_{cp}	Centre of Pressure
x_f	Fin moment arm
SoS	Speed of sound

B Physical Parameters of the HORTON Missile

Symbol	Meaning	Value
ρ_0	Sea Level Air density	$1.23kg/m^3$
ρ	Air Density	$\rho_0 - 0.094h$
d	Reference diameter	$0.2m$
S	Reference area	$d^2/4 = 0.0314m^2$
m	Mass	$125kg$
I_z, I_y	*Lateral Inertia*	$67.5kgm^2$