Improved Detection and Ambiguity Resolution of Low Observable Targets in MPRF Radar

E. J. Hughes and M. B. Lewis
Cranfield University, RMCS
Shrivenham, Swindon, SN6 8LA
ejhughes@iee.org, m.lewis@computer.org

Abstract

This paper presents the results of an investigation into the use of a Multiple Intelligent Software Agent technique as a means of eliminating ghost targets in a MPRF radar system especially when many real targets are present.

1. Introduction

An investigation has been conducted into the formation of ghost targets in range and/or velocity ambiguous radar systems when many targets are on the same azimuth, and extends previous work [1].

The results of the investigation have led to the development of a new PRI schedule strategy that will allow the probability of ghost targets forming tracks to be reduced to a negligible value.

Section 2 introduces medium PRF radar and section 3 details the process of ambiguity resolution. Section 4 investigates the generation of ghosts from multiple targets and section 5 examines the behaviour of ghost targets and their ability to form tracks. Section 6 approximates the expected number of ghosts that will be seen from multiple targets and introduces a process for reducing the probability that the ghosts will form strong tracks. Finally section 7 concludes.

2. Medium PRF Radar

Medium PRF radar systems allow all-round measurements of both the range and Doppler of targets in high clutter environments to be made. Such radars use waveforms that are ambiguous in range, Doppler or both. Existing techniques that resolve these ambiguities require the number of detections input to the ambiguity resolution process to be kept to a small number, as otherwise the number of false correlations ('ghosts') becomes unworkably large.

Another significant issue which affects many look-down airborne radars is the difficulty in distinguishing between unwanted ground moving targets and targets of interest with low closing rates. Commonly these unwanted targets are readily detectable, but must be excluded (for example, by Doppler filtering) to keep the ambiguity resolution problem within bounds.

In commonly used methods of track formation, target returns that cross a detection threshold are taken as 'potential targets'. As the information from the received signal is limited, a false alarm must be treated as a true target, until it can be established as false. A high false alarm rate causes problems with the association of returns with tracks and leads to an excessive number of false tracks being reported with the consequent risk of the tracking system becoming overwhelmed.

3. MPRF Ambiguity Resolution

Medium PRF radar systems were devised as a compromise between Low and High PRF systems and allow all-round measurements of both the range and Doppler of targets in high clutter environments to be made.

A MPRF radar system uses waveforms that are inherently ambiguous in range, Doppler or both. In essence, for each range measurement along an azimuth spoke there are multiple potential targets at ranges given by the expression

$$R = nR^u + \Delta R$$

where R is the true range of the target, R^u is the maximum unambiguous range of the radar at the PRF in use and ΔR is the range as measured by the radar. All ranges are expressed as an integer numbers of range bins.

It is normal to employ several PRIs in order to resolve the ambiguities, thus the range measurements may be generalised as a set of simultaneous equations

$$R = n_i R_i^u + \Delta R_i$$

or in modular arithmetic form as a set of simultaneous congruencies

$$R \equiv \Delta R_i \bmod R_i^u$$

where i represents the various ranges associated with the i^{th} PRI.

3.1 First Chinese Remainder Theorem

Systems of congruencies obey the First Chinese Remainder Theorem [2] which may be stated as: given $n_1, \ldots, n_k$ positive integers which are pairwise coprime (i.e., $\gcd(n_i, n_j) = 1, i \neq j$), then for any given integers $a_1,\ldots,a_k$, there exists an integer x solving the system of simultaneous congruencies

$$x \equiv a_i (\bmod n_i)$$

Furthermore, all solutions x to this system are congruent modulo the product $n = n_1 \ldots n_k$.

The feasible solutions for R_m for a given PRI, in a set of PRIs with unambiguous ranges $\{R^{(1)},\ldots,R^{(N)}\}$, form a set

$$\{R_0^{(n)},\ldots,R_M^{(n)}\} = \bigcup_{m=0}^{M} \{mR^{(n)} + \Delta R^{(n)}\}$$

Since the set represents the set of feasible solutions of R

$$R \in \{R_0^{(n)},\ldots,R_M^{(n)}\}$$

It follows from the uniqueness property of the Chinese Remainder Theorem that

$$\{R\} = \bigcap_{n=1}^{N} \{R_0^{(n)},\ldots,R_M^{(n)}\}$$

The solution as the intersection of the sets of feasible solutions may also be represented graphically in the form of a Venn Diagram [3].

The Coincidence Algorithm, a means of resolving ambiguities, can be derived directly from the above. The solution sets are generated and the intersection is found.

3.2 The Second Chinese Remainder Theorem

The Second Chinese Remainder Theorem [4] states that the number x solves a system of congruencies if

$$x = \sum_{i=1}^{k} a_i e_i, \bmod n$$

and

$$e_i = sn / n_i$$
$$e_i \equiv 1 (\bmod \; n_i)$$
$$n = \prod_{i=1}^{k} n_i$$

The Chinese Remainder Algorithm, as a means of solving congruencies, is a direct application of the Second Chinese remainder Theorem.

4. The Resolution of Ambiguities in the Presence of Multiple Targets

In the case of T targets on the same azimuth then T returns will be taken in each PRF. For the individual targets, t, the set of feasible ranges, $\{R_t\}$, is

$$\{R_t\} = \bigcup_{m=0}^{M} \{m_t^{(n)} R^{(n)} + \Delta R_t^{(n)}\}$$

In the case of two targets and two PRIs then the solution sets are given by

$$\{R_1\} \in \bigcup_{m=0}^{M} \{m_1^{(1)} R^{(1)} + \Delta R_1^{(1)}\}$$
$$\{R_1\} \in \bigcup_{m=0}^{M} \{m_1^{(2)} R^{(2)} + \Delta R_1^{(2)}\}$$
$$\{R_2\} \in \bigcup_{m=0}^{M} \{m_2^{(1)} R^{(1)} + \Delta R_2^{(1)}\}$$
$$\{R_2\} \in \bigcup_{m=0}^{M} \{m_2^{(2)} R^{(2)} + \Delta R_2^{(2)}\}$$

Since it is not possible to associate the individual returns measured with a given PRI with the individual targets the solution pair must exist in the union of the feasible solution sets of returns for each PRI which gives

$$\{R_1, R_2\} \in \bigcup_{m=0}^{M} \{m_1^{(1)} R^{(1)} + \Delta R_1^{(1)}\} \cup \bigcup_{m=0}^{M} \{m_2^{(1)} R^{(1)} + \Delta R_2^{(1)}\}$$
$$\{R_1, R_2\} \in \bigcup_{m=0}^{M} \{m_1^{(2)} R^{(2)} + \Delta R_1^{(2)}\} \cup \bigcup_{m=0}^{M} \{m_2^{(2)} R^{(2)} + \Delta R_2^{(2)}\}$$

The unique solution must exist in the intersection of the two feasible solution sets. Using the Set form of the First Chinese Remainder Theorem and the Distribution Law for Set Union an expression in set form for the solution pairs is found to be

$$\{R_1, R_2\} \in \left\{ \begin{array}{l} \bigcup_{m=0}^{M} \{m_1^{(1)} R^{(1)} + \Delta R_1^{(1)}\} \cap \bigcup_{m=0}^{M} \{m_1^{(2)} R^{(2)} + \Delta R_1^{(2)}\}, \\ \bigcup_{m=0}^{M} \{m_2^{(1)} R^{(1)} + \Delta R_2^{(1)}\} \cap \bigcup_{m=0}^{M} \{m_1^{(2)} R^{(2)} + \Delta R_1^{(2)}\}, \\ \bigcup_{m=0}^{M} \{m_1^{(1)} R^{(1)} + \Delta R_1^{(1)}\} \cap \bigcup_{m=0}^{M} \{m_2^{(2)} R^{(2)} + \Delta R_2^{(2)}\}, \\ \bigcup_{m=0}^{M} \{m_2^{(1)} R^{(1)} + \Delta R_2^{(1)}\} \cap \bigcup_{m=0}^{M} \{m_2^{(2)} R^{(2)} + \Delta R_2^{(2)}\} \end{array} \right\}$$

Solution Set 1

For two targets and two PRIs there are four sets of congruencies to be solved and the First Chinese Remainder Theorem guarantees a solution to all these systems. It is thus not possible to determine unambiguously the range of two targets using only two PRIs.

Since the members of the right hand set are simple combinations it is easy to show that the cardinality of the solution set is T^M where M is the number of PRIs and T is the number of targets.

The RHS set of Solution Set 1 is the solution set for a two target, two PRI system. The LHS set can be rewritten to represent the full solution set

$$\{R_{11}, R_{21}, R_{12}, R_{22}\}$$

Like subscripts represent true targets, dissimilar subscripts represent ghosts.

Since there are T true targets the number of ghosts is found by subtraction

$$T^M - T = T\left(T^{M-1} - 1\right)$$

The number of ghosts generated per scan is invariant and is a function of the number of targets on the same azimuth at any one time, however observation shows that this number is not always apparently present. Non visibility of ghosts may be caused by some ghosts being coincident with true targets or other ghosts. Alternatively the ghosts may lie outside the range of interest.

5. The formation of Ghost Tracks

The Extended Chinese Remainder Theorem states: given positive integers n_1, n_2, n_3 which are coprime (i.e., $\gcd(n_1, n_2, n_3) = 1$) , then there exists a set of integers $\{x^{(1)} \ldots x^{(k)}\}$ solving the system of simultaneous congruencies

$$\begin{aligned}
\{x^{(1)} \ldots x^{(k)}\} &\equiv \{a_1^{(1)} \ldots a_1^{(k)}\} (\operatorname{mod} n_1) \\
\{x^{(1)} \ldots x^{(k)}\} &\equiv \{a_2^{(1)} \ldots a_2^{(k)}\} (\operatorname{mod} n_2) \\
&\vdots \\
\{x^{(1)} \ldots x^{(k)}\} &\equiv \{a_k^{(1)} \ldots a_k^{(k)}\} (\operatorname{mod} n_k)
\end{aligned}$$

Solution Set 1 can be rewritten in matrix form, the arithmetic being mod_n, by using the Second Chinese Remainder Theorem

$$\begin{bmatrix} R_{11} \\ R_{21} \\ R_{12} \\ R_{22} \end{bmatrix} = \begin{bmatrix} \Delta R_1^{(1)} e_1 + \Delta R_1^{(2)} e_2 \\ \Delta R_2^{(1)} e_1 + \Delta R_1^{(2)} e_2 \\ \Delta R_1^{(1)} e_1 + \Delta R_2^{(2)} e_2 \\ \Delta R_2^{(1)} e_1 + \Delta R_2^{(2)} e_2 \end{bmatrix}$$

This can be split into two matrix systems where the system for true targets is

$$\begin{bmatrix} R_{11} \\ R_{22} \end{bmatrix} = \begin{bmatrix} \Delta R_1^{(1)} & \Delta R_1^{(2)} \\ \Delta R_2^{(1)} & \Delta R_2^{(2)} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

and for ghosts is

$$\begin{bmatrix} \Delta R_{21} \\ \Delta R_{12} \end{bmatrix} = \begin{bmatrix} \Delta R_2^{(1)} & \Delta R_1^{(2)} \\ \Delta R_1^{(1)} & \Delta R_2^{(2)} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Since the necessary matrices exist the mapping from the true to ghost targets can be written as

$$\begin{bmatrix} \Delta R_1^{(1)} & \Delta R_1^{(2)} \\ \Delta R_2^{(1)} & \Delta R_2^{(2)} \end{bmatrix}^{-1} \begin{bmatrix} R_{11} \\ R_{22} \end{bmatrix} = \begin{bmatrix} \Delta R_2^{(1)} & \Delta R_1^{(2)} \\ \Delta R_1^{(1)} & \Delta R_2^{(2)} \end{bmatrix}^{-1} \begin{bmatrix} R_{21} \\ R_{12} \end{bmatrix}$$

The above expression shows that ghosts are related to true targets and that the relationship is reciprocal. The implication is that ghost returns form ghost tracks related to the true movement of the target.

Given a set of PRIs, a reasonable estimate for the expected number of ghosts in the region of interest can be calculated (ignoring the possibility of ghost targets falling on top of each other). If our PRI set results in an unambiguous range-Doppler region with on average K range-Doppler cells, and the region of interest has on average Q repeats of the unambiguous region, then we can approximate the probability of ghosts in an M of N system, where M out of N PRIs are required to be coincident.

Assuming T targets are placed in the unambiguous region, and that in the Q repeats of the first PRI/PRF, images of all the targets are present without overlap, then in the second PRI/PRF, the probability of any one target cell overlaying a used cell in the first PRI/PRF is approximately Q/K. Once an overlap has occurred, subsequent PRIs have a probability of achieving an overlap of $1/K$ (as there is no longer a free choice of Q ambiguous regions).

Thus an approximation for the expected number of ghosts in the region of interest, given a probability of detection of 100% is

$$E_G \approx \left(T^M - T\right) \left(\begin{array}{l} \displaystyle\sum_{i=M-2}^{N-2} {}_iC_{N-2} \left(\frac{Q}{K}\right) \left(\frac{1}{K}\right)^i \left(1 - \frac{1}{K}\right)^{N-i-2} \\ \displaystyle + \sum_{i=M-1}^{N-2} {}_iC_{N-2} \left(1 - \frac{Q}{K}\right) \left(\frac{1}{K}\right)^i \left(1 - \frac{1}{K}\right)^{N-i-2} \end{array} \right)$$

The expression can be simplified to

$$E_G \approx \left(T^M - T\right)\left(\begin{array}{l} {}_{M-2}C_{N-2}\left(\dfrac{Q}{K}\right)\left(\dfrac{1}{K}\right)^{M-2}\left(1-\dfrac{1}{K}\right)^{N-M} \\ + \displaystyle\sum_{i=M-1}^{N-2} {}_{i}C_{N-2}\left(\dfrac{1}{K}\right)^{i}\left(1-\dfrac{1}{K}\right)^{N-i-2} \end{array} \right)$$

For a typical airborne fire control system with a 3 of 8 schedule, with K=2000 and Q=100, with T=8 targets, E_G=3 ghosts. With T=10, E_G=19 ghosts and with T=14, E_G=280 ghosts. Thus it is clear that the number of ghosts likely to be present increases very rapidly with only a small increase in the number of strong targets present.

6. Decorrelation of Ghost Tracks

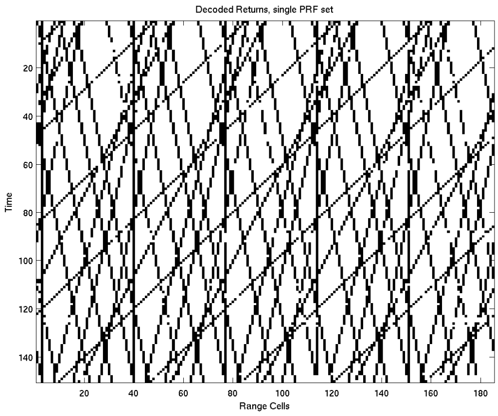

Figure 1. Ghost Tracks from 10 Targets with ambiguities resolved using two PRIs

Figure 1 shows the range-time plot from 10 targets viewed with a 2 PRI system. The 10 targets are: two closing targets with equal velocities; four opening targets with equal velocities; three closing targets with differing velocities plus one stationary target. The PRIs are such that the ambiguity is approximately five times in range.

Severe ambiguity can clearly be observed and as all the ghosts are strong, it is very difficult to determine which tracks are from the real targets. As the probability of detection is 100%, some of the ghost tracks can be identified as they have brief breaks and can be dismissed, but this is a special case. In general, with targets in close formation, the ghosts will appear to move in a very 'target-like' manner.

fragments caused by ghosting. Although approximately the same number of ghosts are present, they occur in a different location for each PRI set, therefore the effect of the PRI changes has been to decorrelate the ghost tracks.

shows the effect of changing the PRI set on a scan to scan basis. Five sets of two PRIs were cycled through. The true tracks are clearly visible against a background of track fragments caused by ghosting. Although approximately the same number of ghosts are present, they occur in a different location for each PRI set, therefore the effect of the PRI changes has been to decorrelate the ghost tracks.

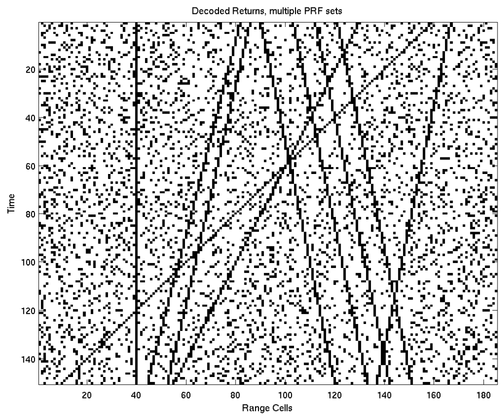

Figure 2. Effect of Scan to Scan PRF Change

Visual inspection indicates a clear set of true target tracks and suggests that a high score would be achieved on such SIAP metrics as accuracy, completeness, continuity and clarity. Unfortunately, the ghost target returns must still be handled by the tracker, and so a tracking system that can handle a very high false alarm rate must be used.

To improve angular resolution, often the M of N processing is performed on a cyclic basis with the last PRI being decoded with the previous N-1, rather than waiting for N new PRIs to be transmitted and decoding as a block. The requirement for a change in the PRI set could make decoding using a rolling PRI scheme more difficult as to allow a rolling system, the PRIs must be intra-set decodable as well as inter-set decodable. Previous work on applying Evolutionary Algorithms to the problem of PRI set optimisation [5] has been reviewed and the technique will be suitable for optimising multiple PRI sets that can be used with a rolling PRI system.

7. Conclusions

This paper has detailed the theory surrounding the decoding of a set of ambiguous range and velocity measurements in the presence of multiple targets. It has been shown that the generation of considerable ghost targets is inevitable, and that their motion is very target-like and ghost returns will correlate scan-to-scan to form tracks of significant length.

A method to help mitigate the problems with the ghost tracks has been presented where an extended PRI set is used to cause the ghost positions to move in a cyclic manner. If the cycle time of the extended PRI set is longer than the association window of the tracker, then the ghost locations appear de-correlated to the tracker and ghost tracks are less likely to be formed.

Acknowledgements

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