CORRELATION OF TARGET TRANSFER FUNCTIONS AND RANGE PROFILES AS A FUNCTION OF ASPECT ANGLE AND RESOLUTION

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Abstract

An experimental rig was constructed in order to investigate the variation of the radar signature of a 1:32 scale model tank with changes in aspect. By illuminating the target with a wideband stepped frequency waveform, a High Range Resolution profile was obtained with an associated frequency spectrum that approximated the Target Transfer Function (TTF). It was found that both TTFs and range profiles de-correlated within a 1° change in target azimuth, more rapidly than previously thought. It was also found that coarser range profiles correlated better than finer ones, but at the loss of a significant amount of distinctive target information. These results have implications for the size of the reference library employed in correlation-matching within the field of Non-Cooperative Target Recognition, and also for the number of different waveforms required to enable matched illumination of a target from all aspects.

1 Introduction

Non-Cooperative Target Recognition (NCTR) aims to recognise targets quickly, at range and within a cluttered environment without any assistance from the target itself. One method of NCTR which has been studied intensely in recent years examines the range profile of a target. A high range resolution radar is used to obtain a profile with a resolution typically of less than one metre. The measured profile is then compared with profiles held in a library in order to identify matches with known target types and so declare the most likely class of target. Furthermore, if one takes the Fourier transform of a range profile one obtains a frequency spectrum of the target. Given a sufficiently wide transmitted bandwidth, this profile approximates to the target transfer function (TTF), which may also be used as a means of target recognition [4].

Another field that employs knowledge of a target's radar signature is that of matched illumination. By understanding where a target's transfer function is greatest, one may concentrate transmitted energy within the associated frequency bands and so maximise the signal energy returned by the target [1]. This results in improved detection ranges and also better target detection within cluttered environments.

One sees that both NCTR and matched illumination systems require prior knowledge of the range profile or TTF which they are trying to detect or recognise. Range profiles and TTFs are however very dependent upon target aspect (among other parameters). An NCTR system needs to store a number of profiles pertaining to the different aspects that a target is likely to be viewed from. The total number of profiles required is clearly dependent upon the extent to which profiles vary with changes in aspect.

This work investigates the de-correlation of range profiles and TTF with changes in target aspect angle through practical measurement and to relate the outcome to theoretical solutions. It also investigates the effect that variations in range resolution have on the correlation of profiles. Measurements will be taken of a 1:32 scale model of a M1A1 Abrams main battle tank. A Vector Network Analyser (VNA) will be used to transmit a wideband millimetric wave stepped frequency waveform, producing a range profile of 1cm resolution (approximately 1/32nd that of current high resolution radars). By measuring range profiles of the target over a number of small changes in azimuth, the rate at which profiles and TTFs de-correlate with each other is assessed.

2 Theory

2.1 Waveform parameters

The range resolution of a radar is given by:

$$\Delta R = \frac{c}{2.B} \tag{1}$$

In which c is the speed of light (=3 x 10^8 m/s) and B is the bandwidth of the transmitted waveform. A stepped frequency continuous wave (SFCW) waveform utilises N discrete frequencies of separation δf and hence:

$$B = (N-1).\delta f \tag{2}$$

The sampled frequency nature of the SFCW waveform invokes range ambiguity resulting in a maximum unambiguous range, R_{mu} of:

$$R_{mu} = \frac{c}{2.\delta f} \qquad (3)$$

2.2 Target signatures

The frequency dependence of the *TTF(ω)* arises from resonances between multiple scatterers on a target or individual structures of dimensions comparable with the wavelength. High range resolution radar (HRRR) seeks to generate a range profile with a fine range resolution. In the limit of infinite bandwidth (infinitesimal range resolution), the *TTF(ω)* is given by the Fourier transform of the range profile. Since the Fourier transform is a linear operation, one would expect both TTF and range profiles to be equally sensitive to aspect changes.

There are two effects which give rise to variations in range profiles with aspect changes. The first is the scintillation of unresolved multiple scatterers within a range resolution cell. The second is the migration of scatterers through range cells. Migration through range cells (MTRC) is avoided if the range of angular rotation is confined to [2]:

$$\delta\varphi \le \frac{\Delta R}{L} \qquad (4)$$

Where L is the cross range dimension of the target. For a given ΔR, MTRC occurs more rapidly if a target's cross-range length is greater and also if the scatterers are generally located towards the outside edges of the target.

2.3 Correlation

Pairs of target profiles (range or frequency), X and Y, are compared using the Pearson Product-Moment Correlation Coefficient, r_{xy} given by [3]:

$$r_{xy} = \frac{\sum\left(X-\overline{X}\right)\left(Y-\overline{Y}\right)}{\sqrt{\left(X-\overline{X}\right)^2\left(Y-\overline{Y}\right)^2}} \qquad (5)$$

Values of +1 indicate perfect correlation, whereas 0 indicates no correlation at all and a value of -1 indicates a perfect negative correlation.

3 Experimental work

An Anritsu ME7808A VNA was operated in the S_{11} mode. The millimetric wave output was connected to a standard gain 20dB horn antenna. A 1:32 scale die cast model of the M1A1 Abrams main battle tank target was placed on an expended polystyrene turntable which was further screened using radar absorbent material (RAM). The reflection from the turntable support was generally 10 to 20 dB lower than the reflection from the tank target. The turntable had an angular resolution of 1^0. The tank target had a footprint of 305 x 114mm and was positioned approximately 1.3m from the horn aperture, which was sufficient to be fully illuminated at all aspects and be in the far field of the horn.

The VNA produces a SFCW waveform which was set up as follows:

Start	90GHz
Stop	105GHz
Number of points	801

Thus with B = 15GHz, N = 801, δf = 18.75MHz, one obtains from equation (1) ΔR = 1cm which is consistent with the resolution of modern HRRR scaled by the same factor as the target i.e. 32. From equation (3) R_{mu} = 8m and is sufficient to avoid any second trace echoes corrupting the target data. The VNA time gating function was used to isolate the return from the target and range profile and/or TTF data recorded. From equation (4) one would expect MTRC to become significant at aspects which exceed 5^0 intervals. A zero degree elevation angle was maintained throughout all the experimental work.

3.1 Correlation of TTF over repeated measurements at head-on aspect

Firstly, the correlation of TTF over successive target placements was measured. The target TTF was measured in the head-on aspect, then the target was rotated to 20^0 and then back to the 0^0 head-on aspect. 5 repeated measurements were made at the head-on aspect in this way. The accuracy of the alignment of the target was estimated at 0.5^0.

3.2 Correlation of TTF over all aspects

The TTF was recorded at 1^0 angular increments over the full 360^0 range of aspect angles. The correlation between pairs of TTFs separated by 1^0 angular displacements was calculated. This was also repeated for pairs of TTFs at between 2 to 15^0 angular increments.

3.3 Correlation of range profiles with aspect angle

The TTF data recorded in section 3.2 was inverse Fourier transformed into the time domain so as to produce range profiles. The range profiles occupied between 30 (head-on) and 12 (side-on) range cells which is far less data than the 801 frequency points used to represent the TTF. The differing length of range profiles invalidates comparisons between range profiles at significantly differing aspect angles. Therefore, only range profiles within 20^0 of head-on (0^0) were considered. At 20^0 the target appears to have a range extent of $305.\cos 20^0$ = 287mm and so the comparison of range profiles over 30 range cells is valid. The correlation between pairs of range profiles separated by between 1 and 10^0 angular increments has been calculated.

3.4 Correlation of range profiles with range resolution

The range profiles generated in section 3.3 were generated at a ΔR = 1cm. Coarser range profiles were generated by averaging neighbouring range cells. Range profiles of ΔR = 2, 3, 4, 5, 6, 10 and 15 cm were generated. The correlation coefficient was calculated between pairs of range profiles seperated in target aspect by 1°, 2^0 and 3^0 for results between 0° and 20° target rotation.

4 Results and discussion

4.1 Repeated measurements at head-on aspect

The 5 repeated tests of section 3.1 give rise to 10 combinations of pairs of TTFs. The correlation between these 10 pairs varies from 0.53 to 0.99. The mean of these 10 correlation coefficients = 0.8 which may be regarded as a high degree of correlation over the estimated angular displacement of 0.5^0. However, in the realms of target identification, if one can only expect to obtain a correlation coefficient on average of 0.8 between two measurements that appear to be taken of nominally the same target aspect, one must temper one's expectation of what may be obtained by correlating a measurement of a particular target aspect with a reference library. The corresponding pairs of range profiles (transformed from the TTF data) exhibited a mean correlation coefficient of 0.86. Although this suggests that range profiles correlate marginally better than TTFs it must be borne in mind that the correlation of the range profiles is on the basis of 30 data points whereas the correlation of TTFs is over 801 data points.

4.2 Correlation of TTF over all aspects

Pairs of TTFs pertaining to measurements taken of target aspects separated in azimuth by increasing amounts were correlated. The expectation was that TTFs with a small separation would correlate better than TTFs with a large separation. Figure 1 displays the correlation coefficient between a TTF measured at a particular target aspect, and the TTF measured after rotating the target by a further 1°. Results are plotted for 0° to 359° target rotation.

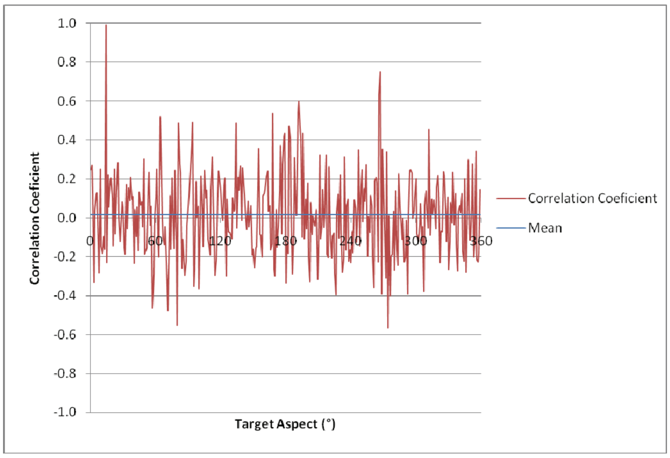

Figure 1: Correlation between TTFs separated by 1°

The mean correlation coefficient was 0.016 indicating that on average, pairs of TTFs measured at target aspects separated by 1° do not correlate with each other. One can see that at certain aspects, weak correlation is exhibited, with correlation coefficients of 0.4 to 0.8 and -0.4 to -0.6. These indicate weak increasing and decreasing linear relationships, but appear to be the exception rather than the norm. A correlation coefficient of 0.99 was obtained when correlating the TTFs associated with 14° and 15° target rotation, indicating that this pair of TTFs is strongly related. However, this result is so rare that it can be discounted as insignificant. The distribution of correlation coefficients is plotted as a histogram in Figure 2.

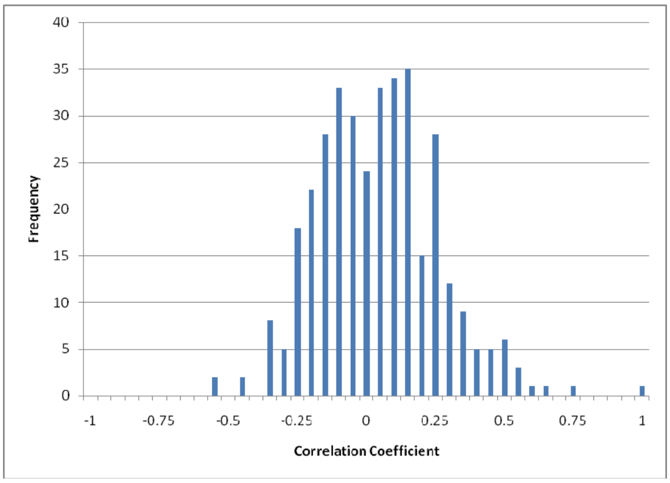

Figure 2: Histogram of correlation coefficients between TTFs separated by 1°

The distribution has a standard deviation of 0.22 and supports the conclusion that TTFs associated with target aspects separated by 1° are unrelated.

Correlation coefficients were calculated in a similar manner for pairs of TTFs separated by an increasing aspect angle and the results are summarised in Table 1.

Angular Separation Between TTFs (°)	Mean Correlation Coefficient
1	0.016
2	0.004
3	-0.010
4	0.017
5	0.002
6	0.003
7	-0.012
8	-0.003
9	0.002
10	-0.003
11	0.004
12	-0.015
13	-0.001
14	0.002
15	-0.002

Table 1: Mean correlation coefficients between TTFs as a function of separation angle.

The results show that in general pairs of TTFs are uncorrelated when measured at target aspects separated by between 1° and 15°. In general there is clearly no similarity or relationship between TTFs associated with target aspects separated by a degree or more. The change in TTF with target aspect occurs much more rapidly than originally expected and TTFs appear to totally de-correlate over 0-1 degrees of target rotation.

4.3 Correlation of range profiles with aspect angle

The range profiles of the tank at 0^0 (head-on), 1^0 and 2^0 occupy 30 range cells and are shown in Figure 3.

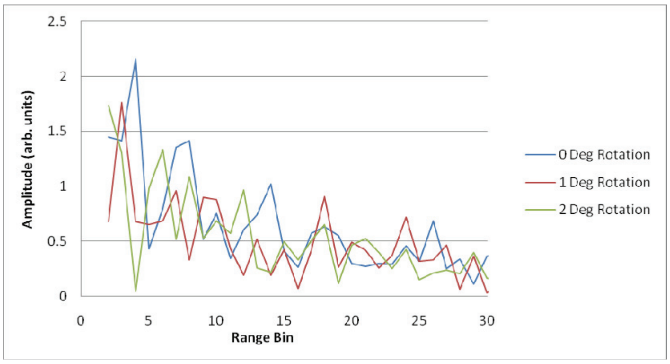

Figure 3: Head-On Aspect Range Profile (1st 30 Range Bins)

The benefit of analysing range profiles as opposed to TTFs is that one might start to associate features of the range profile with physical features on the target. In this case the initial peak might be associated with the front surface of the tank's chassis, with the second peak 4 range bins (40 mm) later perhaps associated with the turret. One observes that the magnitude of the range profile tends to decrease with range, or distance from the antenna. This can largely be attributed to self-shadowing. From Figure 3 one can observe some similarity between the 3 profiles, for instance the common peak at around range bin 18. There are also, however, some notable differences. The initial peak at range bin 4 in the 0° profile quickly becomes almost zero (0.04) after 2° rotation. This illustrates the effect of scintillation.

Table 2 shows the results of correlating pairs of range profiles between 0^0 and 20^0 separated by an increasing amount in azimuth. The correlation coefficients were calculated over the first 30 range bins. The values of 0.3-0.4 (mean 0.37) indicate some weak correlation, but one can see that increasing the angular separation between range profiles does not significantly change the degree to which they correlate. The range profiles all exhibit a degree of similarity, possibly caused by the common trend to reduce in magnitude with range due to self-shadowing effects. These results show that de-correlation occurs much more quickly than predictions based on MTRC – equation (4). This suggests that scintillation effects are more significant and far more sensitive to aspect angle than MTRC.

Angular Separation Between Range Profiles (°)	Mean Correlation Coefficient
1	0.427
2	0.403
3	0.393
4	0.341
5	0.340
6	0.350
7	0.446
8	0.342
9	0.332
10	0.360
Mean	0.373

Table 2: Correlation coefficients between range profiles as a function of target aspect angle separation.

4.4 Correlation of range profiles with range resolution

Range profiles for ΔR = 1, 2, 3, 4, 5, 6, 10, 15cm have been generated and compared. The correlation coefficient was calculated between pairs of range profiles seperated in target aspect by 1°. This was done for results obtained for between 0° and 20° target rotation, so as to avoid the complications with shortening of the range profile with rotation. The mean correlation coeficient was calculated for the various values of ΔR, and the results are plotted in Figure 4.

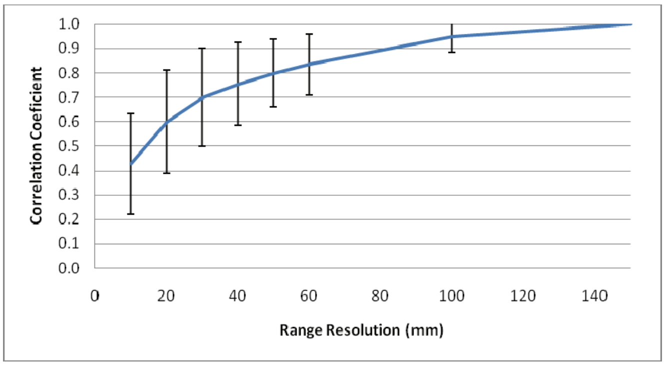

Figure 4: Correlation Coefficient between Range Profiles Separated by 1° in Azimuth.

Each data point has an associated error bar extending to ±1 standard deviation about the mean. One observes that the mean correlation coefficient increases with ΔR whilst the standard deviation tends to decrease. (Standard deviation remains between 0.20 and 0.21 for ΔR between 10-30mm, and drops off thereafter.) When the ΔR is set to 150mm, the correlation coefficient becomes 1. In this case, 2 sets of only 2 data points (range bins) are correlated. The overall tendency in all cases is for the amplitude of the range profile to decrease with range. The correlation coefficient is therefore calculated between 2 linearly decreasing variables. When 2 variables decrease together, the correlation coefficient will equal 1.

This clearly illustrates the limitation in using correlation coefficient as a means of comparison between small samples of data and supports the argument that it is 'easier' to obtain higher values of correlation coefficient between smaller sets of data. Similar correlation statistics were obtained for the correlation between pairs of range profiles separated by 2° and 3° in azimuth.

5 Conclusion

It has been shown that both TTF and range profiles vary significantly with changes in target aspect of more than 1°. It has also been shown that there exists some variation between TTFs and range profiles when the target aspect is changed by less than 1°, indicating that both TTFs and range profiles are very sensitive to small changes in aspect. In the case of small changes in target aspect (<1°), pairs of TTFs were shown to have a mean correlation coefficient of 0.80. Under a similar analysis, pairs of range profiles had a mean correlation coefficient of 0.86. The similarity between these results is to be expected since TTFs and range profiles are related by a Fourier transform. The TTF result is perhaps more significant given the size of its data set. As a correlation coefficient of 0.8 was obtained by comparing TTFs at nominally similar aspect angles, one must moderate one's expectation of what value might be obtained when searching for very close matches. For instance, when employing profile-correlation techniques, the threshold of correlation coefficient which indicates a probable match needs to be established at a value significantly lower than 1, possibly around 0.8, and should be adjusted according to the size of data sets being compared.

This result is, however, quite different when considering larger changes in target aspect. Pairs of TTFs separated by between 1° and 10° target rotation had a mean correlation coefficient of 0.003, indicating that there is no similarity between such pairs. Furthermore, the correlation coefficient between pairs was shown not to significantly deteriorate with increasing separation in aspect. Pairs of TTFs separated by just 1° correlated just as poorly as pairs separated by 10°. Scintillation effects seem to be highly sensitive to aspect angle and tend to dominate over MTRC.

Given such a rapid variation in TTFs, when considering profile-correlation techniques within the realms of NCTR, it may be appropriate to space TTFs within the reference data set at intervals of less than 1°, such as every 0.5° change in aspect. This leads to a requirement to store 82506 TTFs in order to produce a reference set of data covering the upper hemisphere of a single ground target. This number is to be multiplied by the number of expected target types. Alternatively, 82506 different matched waveforms may be required in order to ensure matched illumination of a particular ground target over any aspect. This in turn highlights the signal processing and data storage implications associated with such rapid variations in TTF.

It is also worth noting that whilst the range resolution used in the experimental work here scales accurately to current HRRR systems, the frequency scales to approximately 3GHz on the life sized target. Should the life sized target be illuminated by a higher frequency radar such as a 35 or 94GHz seeker, then one may expect scintillation to be even more sensitive to aspect changes. Correspondingly, TTFs and range profiles will decorrelate even quicker than the results presented here with changes in aspect. The rapid variation in range profiles with aspect reported on here affirms the view of Liao et al [2] who report that
"*For microwave radars, aspect changes of tenths of 1° can cause drastic changes in HRR profiles*".

When considering range profiles, some amount of correlation was evident. Correlating pairs of range profiles separated by between 1° and 10° target rotation produced a mean correlation coefficient of 0.373, indicating a weak relationship. However, as per TTFs, this correlation coefficient did not significantly vary with increasing separation in aspect between pairs. Pairs of range profiles separated by 1° correlated to approximately the same extent as pairs separated by 10°. Degrading the range resolution resulted in superior correlation statistics due to the trend in range profiles to decrease with increasing range on account of self-shadowing. However, the loss of resolution associated with coarser range profiles reduces the separability of different target classes and so is not beneficial to NCTR.

It is a step too far to conclude from this that range profiles correlate better than TTFs, and hence that the time domain may be better to work in from the perspective of non-cooperative target engagement or similar applications. The TTFs each contained many more data points than the range profiles. As mentioned earlier, it is 'easier' to obtain a higher correlation coefficient from two smaller data sets than from two larger ones. The TTFs present a picture of a far higher data dimension that has more scope for variation (including the separability of different target classes) and hence de-correlation than the lower dimension range profiles.

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