A novel mathematical approach for the problem of CFAR clutter model approximation

Giovanni Marino
DISE - Cranfield University
SN6 8LA Shrivenham, United Kingdom
E-mail: g.marino@cranfield.ac.uk

Evan J. Hughes
DISE - Cranfield University
SN6 8LA Shrivenham, United Kingdom
E-mail: e.j.hughes@cranfield.ac.uk

***Abstract*—Automatic Target Recognition (ATR) in Synthetic Aperture Radar (SAR) is a set of techniques which are able to detect, discriminate and classify objects present in the observed scene. Unfortunately the presence of speckle degrades target detection dramatically therefore denoising algorithms are necessary. Moreover sometimes other operations, such as incoherent averaging for instance, are used to increase the Constant False Alarm (CFAR) performance. Any operation as a consequence changes the background clutter model and often it is not possible to describe it in a closed and manageable mathematical form, therefore a direct solution of the Neyman-Pearson problem is not feasible and a suboptimal criterion, such as Exponential or Gamma-distribution clutter model etc., is usually adopted. A consequence of the suboptimal global clutter model choice is the reduction of the information content of the SAR images which can affect heavily classifier performance.**
This paper hence is concerning with a novel mathematical approach for a local approximation of filtered SAR image Cumulative Density Function (CDF) in order to preserve/maximize the information content carried by a SAR/ATR system.

I. Introduction

A Synthetic Aperture Radar/Automatic Target Recognition (**SAR/ATR**) system usually consists of three main actions: detection, discrimination and classification [1], [2] and [3]. First, the entire SAR image is scanned for the target detection stage which requires at least knowledge of the background clutter model. It yields a large number of false alarms in addition to identifying potential targets, therefore it is very important to perform a very effective and efficient detection process.

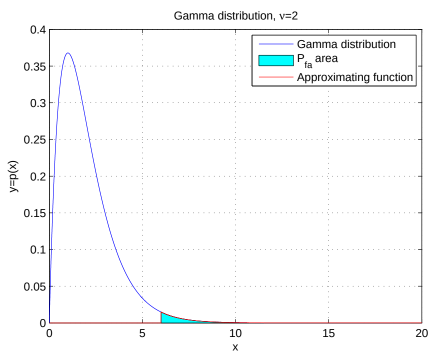

Fig. 1. Exponential approximation of a Gamma-distribution (order parameter $\nu = 2$). The idea is to approximate the P_{fa} area with a function which is easier to manage in the integral in (1).

The outputs of the detection step are then passed to a discrimination stage, which should be able to reject further false targets based on simple properties of potential targets, including both geometrical and electromagnetic effects. Once the detection and discrimination stages have rejected as much clutter as possible, the final stage of an ATR scheme consists of target classification using all the information in the data.
The presence of speckle noise in SAR images affects the discrimination of potential target features, therefore denoising algorithms are usually applied. Moreover other operations, such as incoherent averaging, are performed in order to improve the detection performances. All of the describing operations change the background clutter model (i.e. clutter probability density function, **PDF**), which is crucial to estimate detection process parameters.
Detection can be described in terms of **probability of false alarm** (P_{fa}, also known as **error type I**) which represents the probability that the clutter is considered erroneously a potential target by the detection subsystem, defined as:

$$P_{fa} = \int_t^{\infty} p(x|B)dx \tag{1}$$

where $p(x|B)$ represents the probability that the pixel x is clutter given a clutter model B. The performance of the detector is also described in terms of **probability of detection** (P_d) which is defined as:

$$P_d = \int_t^{\infty} p(x|T)dx \tag{2}$$

where $p(x|T)$ is the likelihood function, i.e. the probability of a data value x when the target is present.
Unfortunately these two quantities are conflicting, therefore an optimization criterion has to be adopted to maximize the P_d with the constraint $P_{fa} \leq \alpha$ (with $0 \leq \alpha \leq 1$).
In radar systems the Neyman-Pearson test is usually considered as the best criterion to overcome the optimization problem and to determine which hypothesis is true (i.e. pixel x is a target or clutter respectively). It states that the target is detected if:

$$\frac{p(x|T)}{p(x|B)} > \tau_{NP} \tag{3}$$

The threshold τ_{NP} is usually selected to give a previously fixed value of $P_{fa} \leq \alpha$ [1] and it can be estimated by inverting

Windows size	λ	k	MSE	T_3	T_4	ϵ_3	ϵ_4
2×2	4.2748	0.9370	$1.4093\ 10^{-10}$	33.6260	45.7100	$1.2777\ 10^{-4}$	$1.2589\ 10^{-7}$
4×4	5.3647	1.4642	$5.3014\ 10^{-11}$	20.0806	24.4402	$1.6594\ 10^{-4}$	$1.2589\ 10^{-7}$
5×5	5.2757	1.6121	$1.5704\ 10^{-11}$	17.4959	20.9140	$2.2762\ 10^{-4}$	$1.5135\ 10^{-5}$

TABLE I

WEIBULL PARAMETERS APPROXIMATION: ϵ_j $(j = 3, 4)$ ENSURES THAT AT THE THRESHOLD T_j THE APPROXIMATING CDF ACTS AS AN LOWER BOUND FOR THE APPROXIMATED CDF

(1) (i.e. fixing the value of P_{fa}, find the smallest value of $t > 0$ that satisfies condition (1)), therefore the knowledge of the clutter model is crucial. Unfortunately in most cases a closed form for the filtered clutter model is not available thereby suboptimal solutions are adopted (e.g. Exponential or Gamma-distributed clutter model [1] [3]).

Fortunately for Detection problems a global clutter model is not necessary, but an approximation of the filtered clutter tails is sufficient because the P_{fa} represents numerically the underlying area of the clutter model tail.

In this paper a novel mathematical approach is introduced to approximate the data output from the denoising process. The idea, as depicted in Fig. 1, can be summarized as follows: the filled area underlying the global clutter model (i.e. Gamma density function with order parameter $\nu = 2$ and scale parameter 1) has to be equal to the underlying area of the approximating function and the initial approximation point has to be equal for both the models. Note that the approximating function should be a function which allows us to compute the Neyman-Pearson threshold through (1) easily.

In section II the novel method is described. Simulations results are discussed in III, whereas the choice of algorithms parameters is analysed in IV. Finally the conclusions are reported in V.

II. METHOD DESCRIPTION

Unfortunately SAR signal processing tends to change the statistics of the background clutter model (as depicted in Fig. 2) and in most cases the outcomes are not computationally feasible in a closed mathematical form. Skolnik in [1] introduces the classical Swerling model II, whereas Oliver in [3] suggests to use K-distribution clutter model and a Gamma approximation for large number of looks L (i.e. the number of radar antenna sub-apertures); Roy in [6] uses a K-distributed form of non-Gaussian clutter. Levanon in [5], Anatassopoulos in [7] however use a global Weibull background clutter model. A local approach of approximating clutter can be more efficient than the classical approach of approximating the clutter distribution (i.e. assuming that outputs are Exponential or Gamma-distributed for instance). A local approach can be made even easier if Cumulative Density Functions (**CDF**s) are considered. CDFs indeed can be mathematically more manageable than Probability Density Functions (**PDF**s). Hence, our problem can be summarized as follows: *'Finding a function which approximates the filtered outputs CDF so that the approximating CDF value corresponding to the threshold is a lower bound for the value of the approximated CDF'*.

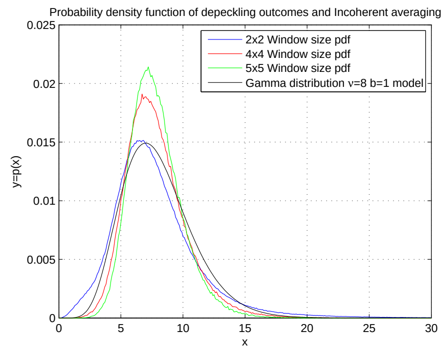

Fig. 2. Probability density functions of outcomes from despeckling and incoherent averaging filters. The simulated data are comapred with a global clutter model: Gamma distribution $\nu = 8$ and scale parameter $b = 1$.

Let (x_0, y_0) and (x_1, y_1) be two points of the CDF which is to approximate and consider, for example, the Weibull CDF:

$$P_{fa} = 1 - \Pr(\mathsf{x} \leq x) = 1 - (1 - e^{-(\frac{x}{\lambda})^k}) = e^{-(\frac{x}{\lambda})^k} \quad (4)$$

where k is the scale parameter and λ is the shape parameter of Weibull clutter model.

The approximating CDF can be computed by solving:

$$\begin{cases} \left(\frac{x_0}{\lambda}\right)^k = -\ln(1 - y_0) \\ \left(\frac{x_1}{\lambda}\right)^k = -\ln(1 - y_1) \end{cases} \quad (5)$$

which determines the values of Weibull parameters $k > 0$ (scale) and $\lambda > 0$ (shape).

As for the choice of the parameters of (5), they will be discussed in the next sections.

III. RESULTS

A set of 1000, 100 *by* 100 pixels, SAR images have been simulated with a clutter model defined by a K-distribution as follows:

$$P(I) = \frac{2}{\Gamma(L)\Gamma(\nu)} \left(\frac{L\nu}{\langle I \rangle}\right)^{(L+\nu)/2} \times I^{(L+\nu-2)/2} K_{\nu-L} \left[2 \left(\frac{\nu L I}{\langle I \rangle}\right)^{1/2}\right] \quad (6)$$

where $L = 1$ is the number of images averaged (number of looks) $\nu = 8$ is the order parameter, $\langle I \rangle = 8$ image intensity mean value, $\Gamma(\cdot)$ is the Gamma Function, K is the modified Bessel Function of second kind. As for the despeckling algorithm, Beltrami flow [22] (single iteration and window size 5 *by* 5) has been adopted. The despeckled

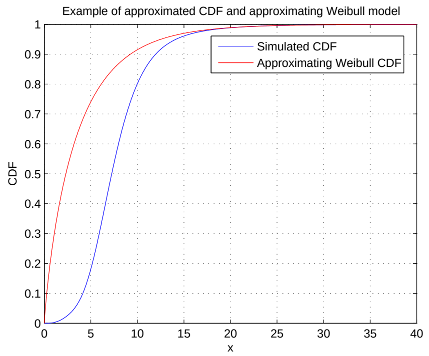

Fig. 3. Example of approximation globally (MSE= 0.0057): The Weibull model (obtained by a filtered images set which has been incoherently averaged with non-overlapped 2 *by* 2 pixels window) with parameters $\lambda = 3.5370$ $k = 0.8677$ (initial point $x_0 = 32$) respectively and the processed images CDF.

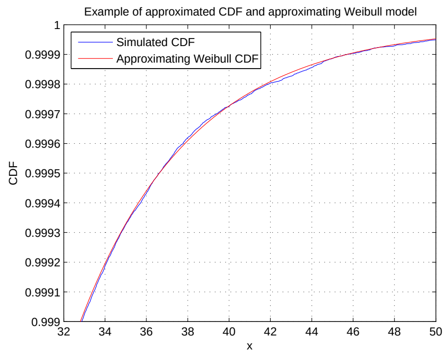

Fig. 4. Example of approximation locally (MSE$\approx 10^{-12}$): The Weibull model (obtained by a filtered images set which has been incoherently averaged with non-overlapped 2 *by* 2 pixels window) with parameters $\lambda = 3.5370$ $k = 0.8677$ (initial point $x_0 = 32$) respectively and the processed images CDF.

images have been then averaged over non-overlapped 2 *by* 2, 4 *by* 4 and 5 *by* 5 pixels windows. Finally the CDFs have been computed.

The computed CDF have been approximated by using (5) with

Windows size	T_3	T_3 (est.)
2×2	32.8833	33.6260
4×4	19.6820	20.0806
5×5	17.1617	17.4959

TABLE II
COMPARISON THRESHOLDS: THE ACTUAL THRESHOLD IS COMPARED WITH THE ESTIMATED ONE FOR $P_{fa} = 10^{-3}$ BY USING THE LOCAL MODEL (5)

following parameters: $x_0 = 15$ ($y_0 = CDF(x_0)$), whereas x_1 is the first value of the approximated CDF such that $|CDF(x_1) - 1| \leq 10^{-4}$ ($y_1 = CDF(x_1)$).

An example is depicted in Fig. 3 and Fig. 4, which represent the same solution seen globally and locally respectively.

The simulations are performed in order to compute the threshold for P_{fa} equal to 10^{-3} and 10^{-4}.

The parameters of approximating CDF are reported in Table

Windows size	T_4	T_4 (est.)
2×2	45.7246	45.7100
4×4	24.4524	24.4402
5×5	20.8721	20.9140

TABLE III
COMPARISON THRESHOLDS: THE ACTUAL THRESHOLD IS COMPARED WITH THE ESTIMATED ONE FOR $P_{fa} = 10^{-4}$ BY USING THE LOCAL MODEL (5)

I: λ and k are the parameters estimated through (5), whereas the Mean Squared Error (**MSE**) between the approximating CDF and the approximated one is computed from the initial point of approximation. T_3 and T_4 however are the thresholds computed (for P_{fa} equal to 10^{-3} and 10^{-4} respectively) by using the complementary of (4):

$$T_s = \lambda [-\ln P_{fa}]^{\frac{1}{k}} \tag{7}$$

As reported in Table II and Table III the thresholds are better estimated for small values of the P_{fa}.

The values of thresholds have been also tested by considering the frequency (i.e. the percentage of pixels) of filtered and averaged images pixels, as previously described, which exceed them (No. of samples: 2×2 $2.5 \cdot 10^6$, 4×4 $6.25 \cdot 10^5$ and 5×5 $4 \cdot 10^5$ respectively). As reported in Table IV, the value of the thresholds produces values of error type I smaller than original P_{fa} (P_{T_3} and P_{T_4} represent the probability of clutter pixels which exceed the thresholds T_3 and T_4 respectively).

The local approximated clutter model has been compared

Windows size	P_{T_3}	P_{T_4}
2×2	$\approx 10^{-4}$	$\approx 10^{-5}$
4×4	$\approx 10^{-4}$	$\approx 10^{-5}$
5×5	$\approx 10^{-4}$	$\approx 10^{-5}$

TABLE IV
ESTIMATED P_{fa} BY USING A LOCAL APPROXIMATED CLUTTER MODEL APPROACH

by two global clutter models: Exponential and Gamma clutter model ($\nu = 8$ and scale parameter 1) respectively.

As for the Exponential clutter model [3], the threshold is computed by considering:

$$T_s = -\sigma_c \ln P_{fa} \tag{8}$$

where σ_c is the mean power of the clutter. The estimated thresholds are reported in Table V.

As for the Gamma-distribution clutter model [16], the thresholds are computed by inverting numerically the following formula:

$$P_{fa} = \sum_{i=0}^{\nu-1} \frac{(\frac{x}{\theta})^i}{i!} e^{(-\frac{x}{\theta})} \tag{9}$$

where $\nu = 8$ and $\theta = 1$ the order parameter and scale parameter of the Gamma-distribution respectively. As a consequence the threshold assume values: $T_3 = 19.2104$, $T_4 = 20.1830$

Windows size	T_3	T_4
2×2	64.9240	86.5654
4×4	34.7892	46.3856
5×5	26.5142	35.3522

TABLE V
ESTIMATED THRESHOLDS BY USING A GLOBAL EXPONENTIAL CLUTTER MODEL

Windows size	P_{T_3}	P_{T_4}
2×2	$\approx 10^{-6}$	$< 10^{-6}$
4×4	$< 10^{-6}$	$< 10^{-6}$
5×5	$< 10^{-6}$	$< 10^{-6}$

TABLE VI
ESTIMATED P_{fa} BY USING A GLOBAL EXPONENTIAL CLUTTER MODEL

respectively.
By comparing Table IV, Table VI and Table VII it is clear that the proposed method is more efficient than other clutter models.

Windows size	P_{T_3}	P_{T_4}
2×2	$\approx 10^{-2}$	$\approx 10^{-3}$
4×4	$\approx 10^{-4}$	$\approx 10^{-4}$
5×5	$\approx 10^{-4}$	$\approx 10^{-5}$

TABLE VII
ESTIMATED P_{fa} BY USING A GLOBAL GAMMA ($\nu = 8$ AND SCALE PARAMETER 1) CLUTTER MODEL

IV. PARAMETERS ANALYSIS

The parameters which have been chosen accurately are the points (x_0, y_0) and (x_1, y_1). We tested the our methods by considering x_1 the first value of the approximated CDF such that $|CDF(x_1) - 1| \leq 10^{-4}$ (i.e. $y_1 = CDF(x_1)$). The value of thresholds seems to be insensitive to the value of the initial point which has been fixed to $x_0 = 2 \cdot mean\ value\ of\ data$. Moreover we suggest to introduce two margins $0 < \eta_j < 10^{-8},\ j = 0, 1$ (subtracted to the actual values y_j) in order to obtain positive errors $\epsilon_i = CDF_{i,approximated} - CDF_{i,approximating}$ (see last two columns of Table I, for $i = 3, 4$). Under this assumption a solution is always found and the estimated thresholds shows that the corresponding estimated P_{fa} is always smaller than the expected one.

V. CONCLUSION

This paper focused on the efficiency of an approximated local clutter model. Three models have been investigated: a local approximation, Exponential and Gamma clutter model. The results confirm that a local approach can be considered more suitable than a global model in terms of CFAR thresholds estimation as well as model fitting of the clutter tail. As a consequence the information content of CFAR input can be preserved/emphasized better (e.g. estimation of SAR/ATR parameters for the discrimination of potential targets such as Mass, Diameter, Rotational inertia, Percent bright CFAR, Standard deviation etc. [3] can be evaluated better) if a approximating local clutter model is adopted.
In this paper a Weibull's model have been adopted, but also Gaussian, Log-normal CDFs can be adopted as approximating CDF if necessary.

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